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Kolloquium über Angewandte Mathematik

Donnerstag, den 15. Januar 2009, 17.15 Uhr, Hörsaal 5

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Convergence analysis for the „Upwind Source at Interfaces“ method for conservation laws with source terms on non-unifom meshes

Zusammenfassung/Abstract

Hyperbolic conservation laws with source terms arise in many applications, especially as simple models for geophysical flows (shallow water equations, nozzle flows, ...), and their numerical approximation leads to specific difficulties. In the context of finite volume/difference schemes, many authors have proposed efficient schemes which can be included into the class of the Üpwind Source at Interfacesmethods. We aim to present a general mathematical formalism for such schemes. We focus on semi-discrete finite volume schemes for scalar conservation laws with a geometrical source term, in the general case of a non-uniform spatial mesh. Typical questions to deal with are the appropriate definitions of consistency and stability, for which we are able to prove an extension of the Lax-Wendroff convergence theorem, and we relate the notion of consistency to the „well-balanced property“. For the (strong) convergence analysis, we concentrate on the discretization of the source term and, under appropriate hypotheses, we prove L^p-error estimates, for $1 \leq p < +\infty$. To improve the numerical accuracy, we formulate two different approaches to treat the source term and we also obtain second order error estimates. Numerical evidence shows that those techniques produce high resolution schemes compatible with the well-balanced methods. In the case of a non-uniform spatial mesh, a suitable notion of (weak) regularity for the cells size is also introduced to obtain optimal results, and we conclude that the same convergence rates hold as for the corresponding homogeneous problems. The general case of non-uniform mesh is particularly interesting, to provide a theoretical framework to the numerical simulation on non-uniform grids (eventually generated by adaptive procedures) of a wide class of mathematical models for real systems. Some examples are discussed, in order to validate the arguments we have proposed.

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