

Complex Functions for Students of Engineering Sciences Homework 7

Problem 1. For $\Gamma = \partial B_\pi(0)$ compute the following curve integrals:

$$(a) \quad \int_{\Gamma} \frac{\sin(z)}{z^2} dz, \quad (b) \quad \int_{\Gamma} \frac{z}{\cos(2z)} dz.$$

Problem 2.

(a) Let $t \in \mathbb{R}$ and $z = e^{it}$. Show that $\cos(t) = \frac{1}{2} \left(z + \frac{1}{z} \right)$ gilt.

(b) Let $B_1(0)$ denote the open unit disk and let $a \in B_1(0) \setminus \{0\}$. Show that

$$\int_0^{2\pi} \frac{1}{1 - 2a \cos(t) + a^2} dt = \int_{\partial B_1(0)} \frac{i/a}{(z - a)(z - 1/a)} dz.$$

(c) Show that

$$\int_0^{2\pi} \frac{1}{1 - 2a \cos(t) + a^2} dt = \frac{2\pi}{1 - a^2}.$$