

Complex functions for students of engineering sciences

Homework 3

Problem 1. Consider the complex cosine-function, defined by

$$\cos(z) = \frac{1}{2}(e^{iz} + e^{-iz}), \quad \text{for } z \in \mathbb{C}.$$

(a) Show that with $z = x + iy$ it holds:

$$\operatorname{Re}(\cos(z)) = \cos(x) \cosh(y), \quad \operatorname{Im}(\cos(z)) = -\sin(x) \sinh(y).$$

$$\text{Reminder: } \cosh(y) = (e^y + e^{-y})/2, \quad \sinh(y) = (e^y - e^{-y})/2.$$

(b) Find all solutions $z \in \mathbb{C}$ of $\cos(z) = \frac{5}{4}$.

$$\text{Hint: } \cosh(\ln(2)) = \frac{5}{4}.$$

Problem 2. Consider the function $w = f(z) := \frac{z}{2} + \frac{2}{z}$, and the sets

$$L = \{z \in \mathbb{C} \mid \operatorname{Re}(z) \leq -2, \operatorname{Im}(z) = 0\}$$

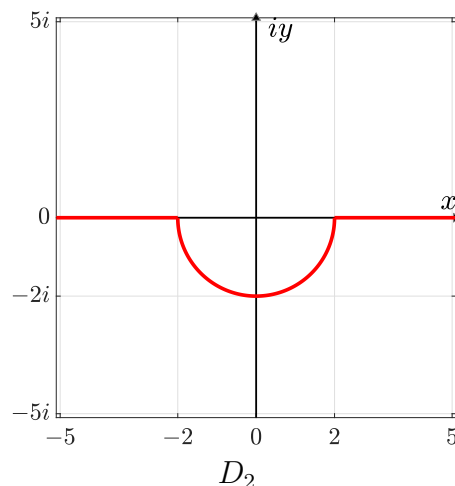
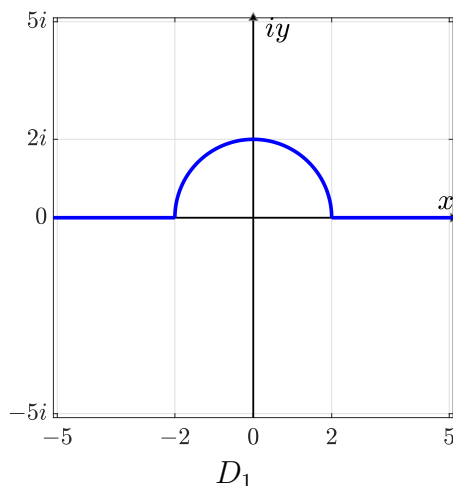
$$R = \{z \in \mathbb{C} \mid \operatorname{Re}(z) \geq 2, \operatorname{Im}(z) = 0\}$$

$$O = \{z \in \mathbb{C} \mid z = 2e^{i\varphi}, \varphi \in (0, \pi)\}$$

$$U = \{z \in \mathbb{C} \mid z = 2e^{i\varphi}, \varphi \in (\pi, 2\pi)\}$$

We let

$$D_1 = L \cup O \cup R, \quad D_2 = L \cup U \cup R.$$



- (a) Determine the images of D_1 and D_2 under f .
- (b) On which of the sets D_1 , D_2 , $D_1 \cup D_2$ is f invertible?
- (c) Compute the inverse function $z = f^{-1}(w)$ for $|z| > 2$.