Fachbereich Mathematik der Universität Hamburg **SoSe 2025**
Prof. Dr. A. Iske, Dr. C. Goetz

Complex functions for students of engineering sciences

Homework 2

Problem 1. Which of the following claims are true, which are false? Give a (short) explanation for your answers.

(a) Claim: *Straight lines* in the complex plane of the form

$$G = \{z \in \mathbb{C} \;\mid\; z = \alpha z_0,\ \alpha \in \mathbb{R}\} \qquad \text{with a fixed } z_0 \in \mathbb{C}$$

are mapped by $f : \mathbb{C} \to \mathbb{C}, \quad f(z) = z^2$, to *beams* of the form

$$H = \{z \in \mathbb{C} \;\mid\; z = \beta w_0,\ \beta \geq 0\ \} \qquad \text{with a suitable } w_0 \in \mathbb{C}.$$

(b) Claim: *Circles* in the complex plane with center $z_0 \in \mathbb{C}$ and radius $R > 0$ are mapped by $f : \mathbb{C} \to \mathbb{C}, \quad f(z) = z^2$, to circles with center z_0^2 and radius R^2.

Problem 2.

(a) Can the set

$$S = \{z \in \mathbb{C} \;\mid\; |\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 1\}$$

be mapped to the set

$$Q = \{z \in \mathbb{C} \;\mid\; \max\{|\mathrm{Re}(z)|,\ |\mathrm{Im}(z)|\} = 1\}$$

by a *linear transformation*? That is, is there a linear map $f : \mathbb{C} \to \mathbb{C}$, such that $Q = f(S)$?

(b) Can the rectangle with vertices

$$z_1 = 1 + \mathrm{i}\sqrt{3}, \quad z_2 = 1 - \mathrm{i}\sqrt{3}, \quad z_3 = -1 - \mathrm{i}\sqrt{3}, \quad z_4 = -1 + \mathrm{i}\sqrt{3}$$

be mapped to the rectangle with vertices

$$w_1 = 1 + \mathrm{i}, \quad w_2 = 1 - \mathrm{i}, \quad w_3 = -1 - \mathrm{i}, \quad w_4 = -1 + \mathrm{i}$$

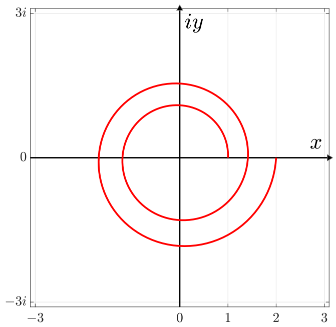
by a linear transformation?

(c) Consider the map $f : \mathbb{C} \to \mathbb{C},\ f(z) = \exp(z)$.

We are looking for a set of the form

$$M = \{z \in \mathbb{C} \ \mid \ z = \alpha z_0,\ \alpha \in [a, b]\}$$

for some $z_0 \in \mathbb{C}$ and some interval $[a, b] \subset \mathbb{R}$, such that the image $\exp(M)$ is a spiral, winding twice around zero (in mathematically positive direction), with starting point 1 and end point 2 (see figure on the right). Find a suitable $z_0 \in \mathbb{C}$ and a suitable $[a, b] \subset \mathbb{R}$ for this.



$\exp(M)$

Problem 3. Consider the equation

$$(z-4)^{20} = z^{20}, \qquad z \in \mathbb{C}.$$

Show that this equation has 19 solutions. Furthermore, show that all of these solutions have real part $\mathrm{Re}(z) = 2$.

Submit by: 02. May 2025