Fachbereich Mathematik der Universität Hamburg **SoSe 2025**
Prof. Dr. A. Iske, Dr. C. Goetz

Complex Functions for students of engineering sciences

Homework 1

Using the *Euler formula* we can represent cosine and sine as

$$\cos(x) = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}\right), \qquad \sin(x) = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}x} - e^{-\mathrm{i}x}\right) \qquad \text{for} \quad x \in \mathbb{R}.$$

Use these representations to proof the following statements.

Problem 1: Show that for all $x \in \mathbb{R}$:

$$\cos^3(x) = \frac{1}{4}\cos(3x) + \frac{3}{4}\cos(x).$$

Problem 2:

(a) For some $n \in \mathbb{N}$, let $a_0, \ldots, a_n$ and $b_1, \ldots, b_n$ be given real numbers. We consider the function $f_n : \mathbb{R} \longrightarrow \mathbb{R}$, defined by

$$f_n(x) := \frac{a_0}{2} + \sum_{\ell=1}^{n} \left[a_\ell \cos(\ell x) + b_\ell \sin(\ell x)\right] \qquad \text{for} \quad x \in \mathbb{R}. \qquad (*)$$

Show that the function f_n can be expressed as

$$f_n(x) = \sum_{k=-n}^{n} c_k \mathrm{e}^{\mathrm{i}kx} \qquad \text{for all } x \in \mathbb{R},$$

with *complex* coefficients $c_{-n}, \ldots, c_n \in \mathbb{C}$. Determine these coefficients

(b) Now let complex numbers $c_{-n}, \ldots, c_n \in \mathbb{C}$, $n \in \mathbb{N}$ be given and consider the function $g_n : \mathbb{R} \longrightarrow \mathbb{C}$, defined by

$$g_n(x) := \sum_{k=-n}^{n} c_k \mathrm{e}^{\mathrm{i}kx} \qquad \text{for } x \in \mathbb{R}.$$

We assume the following:

(i) $c_0 \in \mathbb{R}$;

(ii) $c_\ell = \overline{c_{-\ell}}$, for $\ell = 1, \ldots, n$. I.e., c_ℓ is the complex conjugate of $c_{-\ell}$.

Show that g_n is real and can be expressed in the form $(*)$ with real coefficients $a_0, \ldots, a_n$ and $b_1, \ldots, b_n$.

Submit by: 22. April