

Complex functions for students of engineering sciences

Worksheet 3

Problem 1. Compute the *natural logarithm* of the following complex numbers. What are the *principal values*?

$$(a) \quad z_1 = -\sqrt{3} + i, \quad (b) \quad z_2 = 3e^{i\frac{5\pi}{4}}, \quad (c) \quad z_3 = \left(\frac{\sqrt{3}}{2} + i\frac{3}{2}\right)^4.$$

Compute $\left\{4\operatorname{Log}\left(\frac{\sqrt{3}}{2} + i\frac{3}{2}\right)\right\}$ and compare the result to the result from part (c).

Problem 2.

(a) Determine the Möbius transformation

$$T : \mathbb{C}^* \rightarrow \mathbb{C}^*, \quad T(z) = \frac{az + b}{cz + d}, \quad ad - bc \neq 0,$$

with

$$T(-3) = 0, \quad T(1) = \infty, \quad T(-i) = 1 - 2i.$$

(b) Determine the images of the following sets under the transformation T :

- (i) M_1 : real axis;
- (ii) M_2 : circle with center $1 - i$ and radius 1 .
- (iii) M_3 : circle with center 0 and radius 3 ;

Problem 3. Consider the Möbius - transformation $T(z) = \frac{i(z - 1 - i)}{z + 2 - i}$, and the circle $K = \{z \in \mathbb{C} \mid |z - 2 - i| = 2\}$.

Determine the image of K under T .

Hint: Check $z_1 = 1 + i$ and $z_2 = -2 + i$ for symmetry w.r.t K .