Complex functions for students of engineering sciences Worksheet 2

Problem 1. For each of the following sets, determine its image under the respective map. Draw a sketch of the image, or describe it with words.

(a)

$$D = \{ z \in \mathbb{C} \mid 1 \le |z + i| \le 2, \operatorname{Re}(z) \ge 0, \operatorname{Im}(z) \le -1 \},\$$

$$f: \mathbb{C} \to \mathbb{C}, \quad f(z) = \frac{(z+\mathrm{i})^2}{1+\mathrm{i}}.$$

(b) (problem from an old exam, 5 points)

$$R = \left\{ z = x + iy \in \mathbb{C} \mid |x| \le \frac{\ln(2)}{\pi}, |y| \le \frac{1}{2} \right\},$$
$$f : \mathbb{C} \to \mathbb{C}, \quad f(z) = 2e^{i\frac{\pi}{4}} \cdot e^{\pi z}.$$

(c)

$$V = \{ z = x + iy \in \mathbb{C} \mid 0 < x \le 1, 0 < y \le 1, x^2 + y^2 \le 1 \},\$$

$$f: \mathbb{C} \setminus \{0\} \to \mathbb{C}, \quad f(z) = \frac{1}{z}$$

Problem 2: Determine all complex solutions $z \in \mathbb{C}$ of the following equations.

(a) (problem from an old exam, 3 points) $2e^{3z} - \frac{\sqrt{2}(1+i)}{e^z} = 0$, (b) $z^4 = 8(1+i\sqrt{3})$,