

Complex Functions for students of engineering sciences Worksheet 1

Problem 1: Consider the complex numbers

$$z_1 = 2 + 3i, \quad z_2 = -4 + 5i.$$

Compute the following in *Cartesian coordinates*:

- (a) $z_1 + z_2$, $|z_1 + z_2|$, $2z_1 - 3iz_2$, $2\bar{z}_1 - 3i\bar{z}_2$,
- (b) $z_1 \cdot z_2$, $\bar{z}_1 \cdot \bar{z}_2$, $z_1^2 \cdot z_2^2$, $\operatorname{Re}(z_1^2) \cdot \operatorname{Im}(z_2^2)$,
- (c) $\frac{z_1}{z_2}$, $\frac{\operatorname{Im}(z_1)}{\operatorname{Re}(z_2)}$.

Problem 2: Consider the complex numbers

$$z_1 = 1 + i\sqrt{3}, \quad z_2 = 1 - i\sqrt{3}, \quad z_3 = -1 - i\sqrt{3}, \quad z_4 = -1 + i\sqrt{3}.$$

- (a) Sketch the position of these points in the complex plane and compute their *polar coordinates* ($z = re^{i\varphi}$).
- (b) Compute $z_1 \cdot z_4$ and z_1^7 .
- (c) Compute $\frac{z_1^2 \cdot z_2}{z_3 \cdot \bar{z}_4}$.

Problem 3: Sketch the following sets in the complex plane or describe them with words.

- (a) $M_1 = \{z \in \mathbb{C} \mid |z - 1 + 3i| \leq 2\}$,
- (b) $M_2 = \{z \in \mathbb{C} \mid |z + i| = |z - 3i|\}$,
- (c) $M_3 = \{0\} \cup \{z \in \mathbb{C} \setminus \{0\} \mid \operatorname{Re}\left(\frac{z}{\bar{z}}\right) = 0\}$,
- (d) $M_4 = \{z = re^{i\varphi} \in \mathbb{C} \mid r \in (2, 5), \varphi \in (-\pi/6, \pi/6)\}$.