Exam Complex Functions

August 27, 2025

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

Surname:

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

Problem	points	graded by
1		
2		
3		
4		

$$\sum =$$

Problem 1. (5 points)

Let
$$M = \left\{ z = re^{i\varphi} \in \mathbb{C} \mid r \in [e, e^2], \ \varphi \in \left[-\frac{\pi}{4}, 0 \right] \right\}$$
and
$$f(z) = \mathrm{Log}\left(e^{i \cdot \pi/4} \cdot z^2 \right),$$
where Log denotes the principal branch of the natural logarithm.

(a) Determine the image $f(M)$ of the set M under the function f .

(b) Sketch the sets M and $f(M)$, or describe them with words.

Problem 2. (5 points)

For $z = x + \mathrm{i}y \in \mathbb{C}$ with $x, y \in \mathbb{R}$ let $f : \mathbb{C} \longrightarrow \mathbb{C}$ be given by

$$f(z) \;=\; \frac{1}{2}(x - y)^2 \;+\; \mathrm{i} \cdot \left(\frac{1}{2}x^2 - xy \right).$$

(a) Determine all points in which f is complex differentiable
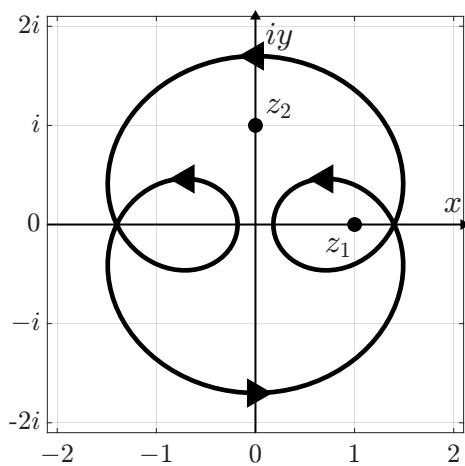
(b) Is f conformal in $z_0 = 0$?
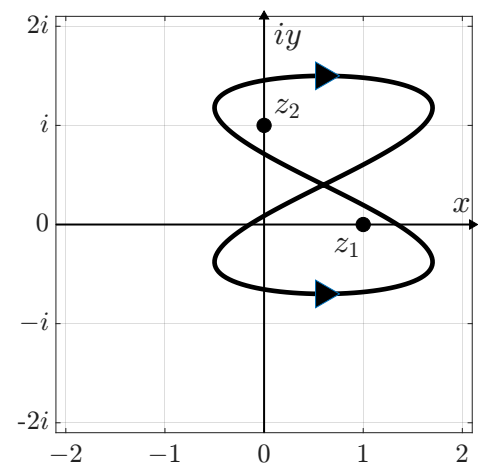
Problem3. (5 points)

Consider the function

$$f(z) \;=\; \frac{1}{(z-1)(z-\mathrm{i})}.$$

(a) Determine and classify all isolated singularities of f.
Compute the corresponding residues.

(b) The following figures show closed curves Γ. We assume that these curves are passed
through once and that the arrows indicate their orientation.

Determine $\displaystyle\int_\Gamma f(z)\,\mathrm{d}z$ for both curves, respectively.

(1)

(2)

Problem 4. (5 points)

(a) Determine the Möbius transformation $T : \mathbb{C}^* \longrightarrow \mathbb{C}^*$ with

$$T(-2i) \;=\; \infty, \qquad T(-i) \;=\; 4, \qquad T(3i) \;=\; 0.$$

(b) Determine the image of the imaginary axis under T.

(c) Let M be the interior of the triangle with corners
$z_0 = -2i, \quad z_1 = -2 + 2i, \quad z_2 = 2 + 2i \quad$ (see figure below).

Which of the following figures shows the image of M under T?
Explain your answer.

M

(1)

(2)

(3)