

## Complex functions for Engineering Students

### Work sheet 6

**Exercise 1:** Calculate the following integrals:

a)  $\oint_{C_1} \frac{e^z}{z} dz, \quad C_1 : [0, 2\pi] \rightarrow \mathbb{C}, \quad C_1(t) = 2 + e^{it},$

b)  $\oint_{C_2} \frac{e^z}{z} dz, \quad C_2 : [0, 2\pi] \rightarrow \mathbb{C}, \quad C_2(t) = 2e^{it},$

c)  $\oint_{C_2} \frac{\pi e^{iz^2}}{(z-i)^2} dz \quad C_2 : [0, 2\pi] \rightarrow \mathbb{C}, \quad C_2(t) = 2e^{it},$

d)  $\oint_{C_3} \frac{z \cos(2z)}{(z - \frac{\pi}{3})^3} dz \quad C_3 : [0, 2\pi] \rightarrow \mathbb{C}, \quad C_3(t) = 1 + e^{it},$

e)  $\oint_{C_4} \frac{z \cos(2z)}{(z - \frac{\pi}{3})^3} dz \quad C_4 : [0, 6\pi] \rightarrow \mathbb{C}, \quad C_4(t) = \frac{1}{2} e^{2it},$

f)  $\oint_{C_5} \frac{z \cos(2z)}{(z - \frac{\pi}{3})^3} dz \quad C_5 : [0, 6\pi] \rightarrow \mathbb{C}, \quad C_5(t) = 1 + 2e^{it},$

g)  $\oint_{C_6} \frac{1}{z^2 + 2z + 10} dz, \quad C_6 : [0, 4\pi] \rightarrow \mathbb{C}, \quad C_6(t) = -3i + 3e^{-it}.$

h)  $\oint_{C_7} \frac{z^2 + 2}{(z^3 - z^2 + z - 1)} dz, \quad C_7 : |z - 0.5| = 1, \text{ traversed once counterclockwise.}$

**Exercise 2:**

Given the function

$$g(z) = \frac{2 + 3z + z^2}{(z^2 + 4)(z^2 - 1)}, \quad f(z) = \frac{1 + z}{z^2(z + i)}, \quad \tilde{f}(z) = \frac{\cos(z) - 2}{z^2} :$$

- How many Laurent series are there for  $g$ ,  $f$  or  $\tilde{f}$  at  $z_0 = 0$ ?
- Determine the Laurent series of the functions  $f$  and  $\tilde{f}$  for the point  $z_0 = 0$  that converges to  $f(2)$  or  $\tilde{f}(2)$  in the neighborhood of  $z^* = 2$ .