

Complex functions for Engineering Students

Sheet 6 (Homework)

Exercise 1:

- a) Let C be the unit circle $|z| = 1$ traversed once in the mathematically positive direction.

(i) Compute
$$\int_C \frac{1}{(e^z - i)} dz.$$

- (ii) For a function analytic on \mathbb{C} , it is given that $|f(z)| = 4$ everywhere on the curve C and $f(0) = 4i$. What must f look like?

- b) Let C be a smooth closed piecewise C^1 curve without double points. When is the integral

$$I(C) := \int_C \frac{z}{z^2 + 1} dz$$

defined?

What values can the integral take if it is defined?

Exercise 2: Determine the Laurent series for the following functions at the point of expansion z_0 , which converges to $f(-3/2)$ at the point $z = -3/2$.

a) $f(z) = z^3 \cos\left(\frac{1}{z}\right), \quad z_0 = 0,$

b) $f(z) = \frac{z^2 + 1}{z^2 + z - 2}, \quad z_0 = 0,$

c) $f(z) = \frac{3}{z^2 + z - 2}, \quad z_0 = 1,$

d) $f(z) = \frac{1}{(z - i)^3}, \quad z_0 = 1 + i.$

Hand in: 24.06.2024 - 30.06.2024