Fachbereich Mathematik der Universität Hamburg **SoSe 2024**
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Complex functions for Engineering Students

Work sheet 5

Exercise 1:

Calculate the following line integrals and sketch the corresponding curves.

a) $\displaystyle\int_{C_1+C_2} |z|\,dz := \int_{C_1} |z|\,dz + \int_{C_2} |z|\,dz,$
C_1 : straight path from -1 to 1,
C_2 : half circle with radius 1 around the origin, from 1 to -1 traversed mathematically positive.

b) $\displaystyle\int_C (1+z)\,dz,$ $C(t) := \cos t + 3i\sin t,\ t \in [-\pi, 0]$ (half ellipse)

c) $\displaystyle\int_c (\bar{z})^2\,dz,$ $c(t) = 2e^{(-1+i)t},\ t \in [0,\ \pi/4],$

d) $\displaystyle\int_C e^{3z}\,dz,$ C : piece of the parabola $\mathrm{Im}\,(z) = \pi\,(\mathrm{Re}\,(z))^2$ that connects the points zero and $1+i\pi$.

Exercise 2:

a) In which area is the Möbius transform $T(z) = \dfrac{az+b}{cz+d}$ angle preserving?

b) Is it possible to map the area

$$M_1 := \{z \in \mathbb{C} : |z| > 1,\ \mathrm{Re}\,(z) > 0,\ \mathrm{Im}\,(z) > 0\}$$

onto the interior of a real triangle via Möbius transform? Here, a real triangle means a triangle whose corners are finite.

c) The mapping $f\ :\ \ z \to e^{\frac{i\pi}{4}}\,\bar{z}$ describes a rotary reflection. Obviously, it does not cause length distortions. The size of the angles is also preserved. f as a transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ is continuously differentiable. Where is f complex differentiable? How does the result compare to the theorem from page 75 from the lecture notes?

Theorem: If $w = f(z)$ is a conformal mapping and continuously differentiable as function $f : \mathbb{R}^2 \to \mathbb{R}^2$, it follows that $f(z)$ is complex differentiable and $f'(z) \neq 0$.

d) The area $G := \{z \in \mathbb{C} : z = re^{i\varphi}\,,\ -\frac{\pi}{8} < \varphi < \frac{\pi}{8}\,,\ 0 < r < 2\}$ is to be transformed onto the interior of the unit circle (bijective and conformal). Why does $z \mapsto \left(\frac{z}{2}\right)^8$ not do that?

Additional exercise: Give a bijective, conformal mapping that achieves this task.

Class: 10.06.24