Fachbereich Mathematik der Universität Hamburg **SoSe 2024**
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Complex functions for Engineering Students

Sheet 5 (Homework)

Exercise 1: Given is a hollow, very long circular cylinder with radius one. Let the top and bottom half be electrically isolated from one another. The upper half has the electric potential $\Phi = 100$ V while the lower one has a potential of $\Phi = -100$ V. With an adequate choice in coordinate systems the intersection of the cylinder with the complex plane yields:

$$\rho^2 \cdot \frac{\partial^2}{\partial \rho^2} \Psi + \rho \cdot \frac{\partial}{\partial \rho} \Psi + \frac{\partial^2}{\partial \alpha^2} \Psi = 0$$

Calculate the potential and field strength inside the cylinder.

Hint : Transform the unit circle onto a sector (e.g. the right half plane). With an adequate transformation, the boundary values for the model are only dependent on the angle. Use the potential equation for polar representation to solve the differential equation in the model domain

$$r^2 \cdot \frac{\partial^2}{\partial r^2} \Psi + r \cdot \frac{\partial}{\partial r} \Psi + \frac{\partial^2}{\partial \phi^2} \Psi = 0$$

while taking the special structure of the boundary values into account. Rewrite the solution for the model domain into cartesian representation and use an inverse transform.

Exercise 2:

a) For $z = x + iy$ let $\bar{z} = x - iy$. Calculate

$$\oint_c \bar{z} \cdot z^{\frac{1}{2}} \, dz$$

along the curve

$$c : [0, \pi] \to \mathbb{C},\ c(t) = 4e^{it} \text{ und } C : [0, \pi] \to \mathbb{C},\ C(t) = 4e^{-it}$$

and confirm that the complex line integral is path-dependent in general.

b) Determine the values of the following line integrals if they exist. The curves are to be traversed once in positive direction.

i) $\displaystyle\int_{C_1} \frac{1}{z-2} \, dz$ $\quad C_1$: Circle with radius 1 around the origin,

ii) $\displaystyle\int_{C_2} \frac{1}{z-2} \, dz$ $\quad C_2$: Circle with radius 2 around the origin,

iii) $\displaystyle\int_{C_3} \frac{1}{z-2} \, dz$ $\quad C_3$: Circle with radius 3 around the origin.

Exercise 3:

a) Given are the functions

$$f_1(z) := \frac{e^z - 1}{e^z + e^{-z}}\,, \qquad f_2(z) := \frac{1}{\ln(3-z)}\,, \qquad f_3(z) := \frac{1}{\ln(\frac{i}{2} - 4 - z)}.$$

Determine the radius of the largest circle around the origin for which the respective Taylor series T_k of f_k at zero converges to f_k (for $k = 1, 2, 3$). Do not explicitly calculate the series.

b) The function $f(z) = \dfrac{1}{z(z^2 - 4z + 13)}$ is to be approximated by a Taylor series at $z_0 := x_0 + iy_0$, $x_0 \in \mathbb{R}^+$, $y_0 \in \mathbb{R}$, that converges to $f(z)$ in the circular disk $|z - z_0| < |z_0|$ (at minimum). How must z_0 be chosen so that x_0 becomes as large as possible.

Hand in: 10.- 16.06.24