

Complex functions for Engineering Students

Work Sheet 3

Exercise 1:

Let $\ln(z)$ be the principal value for the complex logarithm. Given the following complex numbers

$$z_1 = \sqrt{2}(-1 + i), \quad z_2 = 3i, \quad z_3 = -4i.$$

a) Calculate the cartesian representations for

$$z_4 := z_1 \cdot z_2, \quad z_5 := \frac{z_1}{z_2}, \quad z_6 := z_1 \cdot z_3, \quad z_7 := \frac{z_1}{z_3}.$$

b) Calculate $\ln(z_k)$, $k = 1, 2, \dots, 7$.

c) Compare

$$\ln(z_1 \cdot z_k) \text{ with } \ln(z_1) + \ln(z_k)$$

and

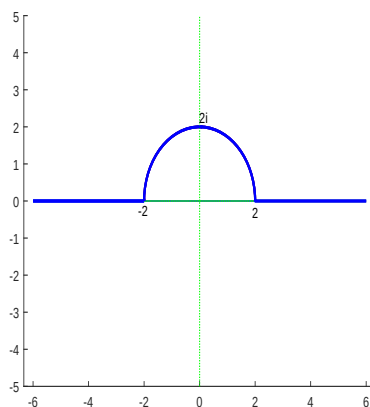
$$\ln\left(\frac{z_1}{z_k}\right) \text{ with } \ln(z_1) - \ln(z_k) \text{ for } k = 2, 3.$$

d) For what complex numbers do the rules for $\mathbb{R}$ apply:

$$\ln(a \cdot b) = \ln(a) + \ln(b), \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) ?$$

Exercise 2: Let the two sets D_1 and D_2 be

$$D_1 := \{x \in \mathbb{R} : -\infty < x \leq -2\} \cup \left\{z \in \mathbb{C} : z = 2e^{i\phi}, \phi \in]0, \pi[\right\} \cup \{x \in \mathbb{R} : 2 \leq x < \infty\},$$



and

$$D_2 := \{x \in \mathbb{R} : -\infty < x \leq -2\} \cup \{z \in \mathbb{C} : z = 2e^{i\phi}, \phi \in]\pi, 2\pi[\} \cup \{x \in \mathbb{R} : 2 \leq x < \infty\}.$$

Determine the images of D_1 and D_2 for the mapping $f(z) = \frac{2}{z} + \frac{z}{2}$.

On which of these sets D_1 , D_2 , $D_1 \cup D_2$ is f invertible?

Exercise 3:

- a) (i) How many solutions does the equation $(z - 2i)^{10} = z^{10}$ have?
(ii) Show that all solutions for the equation from i) lie on the line $\operatorname{Im}(z) = 1$.
- b) How many solutions does the equation $(z - 2i)^i = z^i$ have?