Fachbereich Mathematik der Universität Hamburg **SoSe 2024**
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Complex functions for Engineering Students
Work sheet 1

Excercise 1:

a) Rewrite these complex numbers into polar representation ($z = re^{i\phi}$) and draw a sketch of them in the complex plane.
$z_0 = -4, \quad z_1 = \sqrt{8}(-1-i), \quad z_2 = -4i, \quad z_3 = \sqrt{8}(1-i), \quad z_4 = 4,$
$\tilde{z}_k = i^k, \quad k \in \mathbb{Z}.$

b) Rewrite these complex numbers into cartesian representation ($z = x + iy$).
$z_5 = 3\,e^{i\frac{\pi}{3}}, \quad z_6 = 2e^{i\frac{-\pi}{6}}, \quad z_7 = 2e^{i\frac{-13\pi}{6}}.$

Excercise 2: Let $z_1, \ldots, z_6$ be defined as in exercise 1. Calculate the Cartesian representations of the following complex numbers.

$$\mathrm{Re}\,(z_1), \quad \mathrm{Im}\,(z_1), \quad \mathrm{Re}\,(z_3), \quad \mathrm{Im}\,(z_3)\,, \quad z_1 + z_3\,, \quad z_1 - z_3\,,$$
$$2z_5 + \sqrt{8}z_3\,, \quad \bar{z}_1\,, \quad z_1 \cdot \bar{z}_1\,, \quad z_1 \cdot z_2\,, \quad (z_6)^2 \cdot (z_5)^4\,, \quad \frac{z_5}{z_6}\,.$$

Excercise 3: Characterize these subsets of the complex plane by sketch or explanation:

$$\begin{aligned} M_1 &= \{z \in \mathbb{C} \mid |z + 4 - 3i| \le 5\}, \\ M_2 &= \{z \in \mathbb{C} \mid |z - i| = |z - 2 - i|\}\,, \\ M_3 &= \{z \in \mathbb{C} \mid z + \bar{z} = 2\}\,, \\ M_4 &= \{0\} \cup \left\{z \in \mathbb{C} \setminus \{0\} \mid \mathrm{Re}\left(\frac{z}{\bar{z}}\right) = 0\right\}. \end{aligned}$$

Excercise 4: Describe the following subsets of the complex number plane using formulas similar to those in task 3.

M_5: strip parallel to the imaginary axis with a width of 4, symmetric to $z_0 = 1 + i$, including the boundary.

M_6: circular disk around the origin with inner radius 1 and outer radius 3, without boundary.

M_7: circular disk (punctured disk) around the origin with inner radius 0 and outer radius 3, without boundary.

M_8: sector between the lines $\mathrm{Re}\,(z) = \mathrm{Im}\,(z)$ and $-\mathrm{Re}\,(z) = \mathrm{Im}\,(z)$ in the upper half-space, without boundary.

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