Mathematics Department
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SuSe 2024

Exam: Complex Functions

26.August 2024

Please mark each page with your name and your matriculation number.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exams beginning in retrospect.

(Signature)

Task no.	**Points**	**Evaluator**
1		
2		
3		
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Task 1)   [5 points]

Let i be the imaginary unit and R the rectangle

$$R := \left\{ z \in \mathbb{C} \mid z = x + iy,\ x, y \in \mathbb{R},\ |x| \leq \frac{\ln(2)}{\pi},\ |y| \leq \frac{1}{2} \right\}.$$

Determine the image of R under the mapping

$$f : \mathbb{C} \to \mathbb{C}, \qquad f(z) := 2e^{i\frac{\pi}{4}} \cdot e^{\pi z}.$$

Make a sketch of the image or describe the image in words.

Task 2) [6 points]

a) Determine a Möbius transform

$$T : \mathbb{C}^* \to \mathbb{C}^*, \quad T(z) := \frac{az+b}{cz+d}$$

that satisfies

$$T(3) = 0, \qquad T(0) = -6, \qquad T(-1) = \infty.$$

b) Which generalized circles in $\mathbb{C}$ are mapped onto straight lines by T?

c) Determine the images of the following generalized circles under T from part a):

(i) $K :=$ real axis,

(ii) $\hat{K} := \{z \in \mathbb{C} \mid |z-1| = 2\}$,

(iii) $\tilde{K} :=$ imaginary axis.

Task 3) [6 points]

Given

$$f(z) = \frac{z+1}{z^3 + 3z^2}.$$

a) Determine and classify all isolated singularities of f.

b) Compute the residues of f at all isolated singularities.

c) How many different Laurent series of f exist for the expansion point $z_0 = 1$? Specify the rings in which the Laurent series converge to f.

d) Compute $\oint_{C_k} f(z)\,dz$ for $k = 1, 2$.

(i) $C_1 : [0, 2\pi] \to \mathbb{C}$, $C_1(t) = 3 + 2e^{it}$,

(ii) $C_2 : [0, 2\pi] \to \mathbb{C}$, $C_2(t) = 2e^{it}$.

Task 4) [3 points]

Given the function $f : \mathbb{C} \setminus \{1\} \to \mathbb{C}$,

$$f(z) := \frac{z}{1 - \bar{z}}$$

and the curve

$$c : [0, \frac{\pi}{2}] \to \mathbb{C}, \quad c(t) = 1 + 2e^{it}.$$

Compute the curve integral

$$I_C := \int_c f(z)\, dz.$$