Fachbereich Mathematik der Universität Hamburg **SoSe 2023**
Prof. Dr. J. Struckmeier
Dr. K. Rothe, E. Ficola

Complex functions for Engineering Students

Exercise class 5

Exercise 1:

For $f : \mathbb{C} \to \mathbb{C}$ mit $f(z) = z^3$ compute

a) $A := \frac{1}{2}\left(f_x(z_0) - i f_y(z_0)\right)$ and

b) $B := \frac{1}{2}\left(f_x(z_0) + i f_y(z_0)\right)$.

Compare the results with partial derivatives of f with respect to the independent variables z und $\bar{z}$, that is with

$$\frac{\partial f}{\partial z}, \quad \frac{\partial f}{\partial \bar{z}} .$$

For this apply formally the usual rules of derivation on the real field.

Exercise 2:

a) Decide (with justifications) whether

(i) $f(z) = z^2 + \bar{z}^2 + 4i \cdot \mathrm{Re}(z)\mathrm{Im}(z) + i$ is holomorphic,

(ii) $g(z) = \mathrm{Re}(e^z)$ is holomorphic,

(iii) $\mathrm{Re}(z^{10} + \sin^7 z)$ is harmonic.

b) Let the function

$$v(x,y) = 2xy - 6y + e^x \sin y$$

be given.

(i) Show that v is harmonic.

(ii) For $v(x,y)$ determine a function $u(x,y)$ such that the function $f(z) = u(x,y) + iv(x,y)$ with $z = x + iy$ is holomorphic.

Exercise 3:

Let the curves $c_1(t) = it$ and $c_2(t) = e^{it}$ be given, in each case for $0 < t < \pi$.

a) Draw the curves c_1 and c_2 in the z-plane and determine their intersection point with intersection angle.

b) Into which image curves of the w-plane are c_1 and c_2 mapped into by the principal value of $w = \ln z$? Check whether the intersection angle of the curves and the local length ratio are preserved.

Dates of classes: 5.6. - 9.6.