Complex functions for Engineering Students

Homework 5

Exercise 1:

a) Draw the line $G = \{z \in \mathbb{C} \mid z = -1 + it , \ t \in \mathbb{R}\}$ and the circumference $K = \{z \in \mathbb{C} \mid |z - 2| = \sqrt{5}\}$ and compute the two points z_1 and z_2 which lie symmetrically to G and K.

b) Determine all conformal functions

$$T(z) = \frac{az + b}{cz + d}$$

with $T(z_1) = 0$ and $T(z_2) = \infty$.

c) Draw the images of G and K under T, in case it holds still $T(-1) = -1$.

Exercise 2:

Consider the half-plane E lying to the right of the line $G = \{z \in \mathbb{C} \mid z = -1 + it, t \in \mathbb{R}\}$ and outside the circular disc $K = \{z \in \mathbb{C} \mid |z - 2| \leq \sqrt{5}\}$.

Compute a function harmonic in E and such that it has value 1 on the boundary of K and 0 on G.

Hint: Transform the problem as given in Exercise 1, solve the conformal transformed problem in polar coordinates and then transform back.

Hand in until: 9.6.