Fachbereich Mathematik der Universität Hamburg **SoSe 2023**
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Complex functions for Engineering Students

Exercise class 3

Exercise 1:
Determine the image of

$$K := \left\{ z \in \mathbb{C} \mid 0 \le \mathrm{Re}(z)\,,\ 0 \le \mathrm{Im}(z)\,,\ \mathrm{Re}(z)^2 + \mathrm{Im}(z)^2 \le 1 \right\}$$

under the mapping defined by $f(z) = ((1+i)z)^2$.

Exercise 2:

For the exponential function exp determine the images of the following sets

a) $D_1 = \left\{ z \in \mathbb{C} \mid 0 \le \mathrm{Re}(z) \le \ln(10)\,,\ -\frac{\pi}{2} \le \mathrm{Im}(z) \le \frac{\pi}{2} \right\}$,

b) $D_2 = \{ z \in \mathbb{C} \mid \mathrm{Re}(z) \le 0\,,\ 0 \le \mathrm{Im}(z) < \pi \}$,

c) $D_3 = \{ z \in \mathbb{C} \mid -\pi < \mathrm{Im}(z) < \pi \}$.

Exercise 3:

a) Given $z_1 = 2 + \dfrac{\pi i}{3}$ and $z_2 = -1 + \dfrac{2\pi i}{3}$, compute

$$\exp(z_1)\,,\ \exp(z_2) \quad \text{and} \quad \exp(z_1 + z_2)$$

in Carthesian coordinates and verify with such example the validity of the functional equation for the exp function in $\mathbb{C}$:

$$\exp(z_1) \cdot \exp(z_2) = \exp(z_1 + z_2)\,.$$

b) For the principal value of the complex logarithm log, with $z_1 = -1 - i\sqrt{3}$ and $z_2 = -2i$ one compute

$$\log(z_1)\,,\ \log(z_2) \quad \text{and} \quad \log(z_1 z_2),$$

and check with such example whether for the principal part it holds:

$$\log(z_1) + \log(z_2) = \log(z_1 z_2)\,.$$

Dates of classes: 2.5.-5.5.