Fachbereich Mathematik der Universität Hamburg **SoSe 2023**
Prof. Dr. J. Struckmeier
Dr. K. Rothe, E. Ficola

Complex functions for Engineering Students

Exercise class 2

Exercise 1:

Draw the following sets of points in the complex plane:

a) $\{z \in \mathbb{C} : |3z + 6 - i| = 9\}$,

b) $\{z \in \mathbb{C} : \text{Re}(z) \le \text{Im}(z)\}$,

c) $\{z \in \mathbb{C} : \text{Re}((1 - i)z) = 2\}$,

d) $\{z \in \mathbb{C} : \pi \le \arg(z) \le 3\pi/2 \, , \; 4 \le |z| \le 5\}$.

Exercise 2:

a) For $z \in \mathbb{C}$ consider the polynomial $p(z) := a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ with real coefficients $a_0, \dots a_n$.

Show that if $z_0 \in \mathbb{C}$ is a root of p, then also $\bar{z}_0$ is a root of p.

b) Prove that the circle $|z - z_0| = r$ in the complex plane has the following representation

$$z\bar{z} - z\bar{z}_0 - z_0\bar{z} + z_0\bar{z}_0 = r^2 \quad \text{with} \quad z, z_0 \in \mathbb{C}.$$

c) Determine the curve described by

$$z\overline{z} = (4 - 3i)\overline{z} + (4 + 3i)z + 144.$$

Exercise 3:

Analyze the convergence of the sequence

$$z_0 = 3 \, , \quad z_{n+1} = \frac{3 - 2i}{4}\,(1 + 2i + z_n)$$

and if possible determine its limit value.

Dates of classes: 17.4. - 21.4.