Department of Mathematics
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WiSe 2023/24

Mathematics IV Exam
(Module: complex Funktions)

4. March 2024

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in **BLOCK CAPITALS** each in the following designated fields. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

(Signature)

Task no.	Points	Examiner
1		
2		
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4		
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6		

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Exercise 1: (2 points)

For the exponential function exp compute the image of the set D with drawing

$$D = \left\{ z \in \mathbb{C} \mid 0 \leq \mathrm{Re}(z) \leq \ln(2)\,,\ 0 \leq \mathrm{Im}(z) \leq \frac{\pi}{2} \right\}.$$

Exercise 2: (2+1+2 points)

a) Determine the two points z_1 and z_2 lying symmetrically with respect to the imaginary axis and to the circumference $K = \{z \in \mathbb{C} \mid |z+5| = 3\}$.

(Hint: z_1 and z_2 lie on the real axis.)

b) Compute all the Möbius transformations T such that it holds

$$T(z_1) = 0\,, \quad T(z_2) = \infty\,.$$

c) Determine the image of K under T, in case additionally it holds $T(-2) = i$.

Exercise 3: (2+1 points)

For the function f given by

$$f(z) = z^2 - \bar{z}^2 + 2 \cdot \mathrm{Re}(z^2) + 3$$

decide (with justification) whether

a) $f(z)$ is holomorphic and

b) $\mathrm{Re}(f(z))$ is harmonic.

Exercise 4: (1+1 points)

Compute the line integral

a) $\displaystyle\int_c \frac{1}{z^2}\,dz$ for $c(\varphi) = e^{i\varphi}$ with $\pi \leq \varphi \leq \dfrac{3\pi}{2}$,

b) $\displaystyle\oint_{|z-i|=2} \frac{e^z}{(z+1)^5}\,dz$ with positively oriented path of $|z-i| = 2$.

Exercise 5: (2+2+1+1 points)

Let the function f defined by $f(z) = \dfrac{12}{z^2+4}$.

a) Determine the type of all singularities of f and compute the corresponding residues.

b) Determine and draw the convergence domain of all power series expansions of f around $z_0 = i$.

c) Provide the complex partial fraction decomposition of f.

d) Compute $\displaystyle\int_{-\infty}^{\infty} \frac{12}{x^2+4}\, dx$.

Exercise 6: (2 points)

Let the function f with $f(z) = \dfrac{\cos(z) - 1}{z^3}$ be given.

For f indicate the Laurent series development converging around $z_0 = 0$, classify all the singularities and determine the corresponding residues.