Department of Mathematics
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SoSe 2023

Mathematics IV Exam
(Module: complex Funktions)

5. September 2023

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in **BLOCK CAPITALS** each in the following designated fields. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

(Signature)

Task no.	Points	Examiner
1		
2		
3		
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6		

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Exercise 1: (2 points)

Determine the image of

$$K := \left\{ z \in \mathbb{C} \,|\, -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4} \,,\; |z| \leq 1 \right\}$$

under the mapping defined by $f(z) = z^2 + 1$ and draw it.

Exercise 2: (2+1+2 points)

Let a Möbius transformation $w = T(z)$ be given with

$$T(-2i) = 0 \quad \text{and} \quad T(0) = -3.$$

a) Determine T such that the lower half plane $\mathrm{Im}(z) \leq 0$ is mapped onto the circular disk

$$K := \{w \in \mathbb{C} \mid |w| \leq R\}.$$

(Hint: $z_1 = -2i$ and $z_2 = 2i$ lie symmetrically with respect to the real axis.)

b) Compute the radius R of the disk K.

c) Calculate T.

Exercise 3: (1+2 points)

Let the function defined by $u(x, y) = y^2 + 3x - x^2 + e^{-x} \cos(y)$.

a) Show that u is harmonic.

b) Construct a function $v(x, y)$ such that the function $f(z) = u(x, y) + iv(x, y)$ with $z = x + iy$ is holomorphic.

Exercise 4: (1+1 points)

Compute the line integrals

a) $\displaystyle\int_c \frac{1}{z}\,dz$ for $c(\varphi) = e^{i\varphi}$ with $0 \le \varphi \le \dfrac{\pi}{2}$,

b) $\displaystyle\oint_{|z-1|=1} \frac{\sin(z)}{\left(z-\frac{\pi}{2}\right)^3}\,dz$ with positively oriented path of $|z-1| = 1$.

Exercise 5: (3+2+1 points)

Let the function f defined by $f(z) = \dfrac{3}{z-2} + \dfrac{6}{z-5}$.

a) For the development point $z_0 = 2$ one compute all the power series expansions of f and draw their convergence domains.

b) Determine the type of all singularities of f and give the corresponding residues.

c) Compute $\displaystyle\oint_{|z-3|=3} f(z)\,dz$ for the simple curve $|z-3| = 3$ running in the positive mathematical orientation.

Exercise 6: (2 points)

Let the function f be given with $f(z) = (z-2)^3 \exp\left(\frac{1}{z-2}\right)$.

For f determine the convergent Laurent series expansion around $z_0 = 2$, classify all the singularities and determine the corresponding residues.