

Complex functions for Engineering Students

Sheet 7 (Homework)

Exercise 1: Given $f(z) = \frac{z+i}{(z-2)^2(z^2+1)}$.

- a) Find all isolated singularities of f and classify them.
- b) Calculate the residues of all isolated singularities of f .
- c) Determine the complex partial fraction for f .

Exercise 2:

Calculate the following integrals or their (Cauchy) principal values with the use of the residue (see lecture notes pages 150-152).

a)

$$\int_{-\infty}^{\infty} \frac{x \sin(\omega x)}{x^2 + 4} dx \quad \omega > 0.$$

b)

$$\int_0^{\pi} \frac{1}{1 + \sin^2 \varphi} d\varphi.$$

c)

$$\int_0^{\infty} \frac{\sqrt{x}}{x^3 + 2x^2 + 2x} dx.$$

Also give cartesian representation for the result of the last (real) integral.

Exercise 3:

Calculate the Fourier transforms of the following functions:

$$\text{a) } f(t) = \begin{cases} -1 & \text{für } t \in (-a, 0) \\ 1 & \text{für } t \in (0, a) \\ 0 & \text{sonst.} \end{cases}$$

$$\text{b) } f(t) = \frac{1}{4 + t^2}.$$