

Complex functions for Engineering Students

Solutions for work sheet 6

Given the function

$$g(z) = \frac{2 + 3z + z^2}{(z^2 + 4)(z^2 - 1)}, \quad f(z) = \frac{1 + z - z^2 + iz^3}{z^2(z + i)}, \quad \tilde{f}(z) = \frac{\cos(z) - 2}{z^2} :$$

- a) Determine and classify all isolated singularities of g , f and $\tilde{f}$.
- b) Calculate the integrals of f along the circles K_m , $m = 1, 2, 3$ which are traversed once in positive direction.
- $$K_1 := \left\{ z \in \mathbb{C} : |z - i| = \frac{1}{2} \right\}$$
- $$K_2 := \left\{ z \in \mathbb{C} : |z| = \frac{1}{2} \right\}$$
- $$K_3 := \left\{ z \in \mathbb{C} : |z + i| = \frac{1}{2} \right\}.$$
- c) How many Laurent series are there for g , f or $\tilde{f}$ at $z_0 = 0$?
- d) Determine the Laurent series of the functions f and $\tilde{f}$ for the point $z_0 = 0$ that converges to $f(2)$ or $\tilde{f}(2)$ in the neighborhood of $z^* = 2$.

Hint: polynomial long division should be used!