

Complex functions for Engineering Students

Solutions for sheet 6 (Homework)

Exercise 1:

Calculate the following path integrals (if they are defined). The curves are to be traversed once (positively).

$$\text{a) } \int_{C_k} \frac{4e^{\pi z}}{(z-2i)} dz \quad k = 1, 2, 3, \quad C_k : |z| = k,$$

$$\text{b) } \int_{C_k} \frac{e^z}{(z-2i)^5} dz \quad k = 1, 2, \quad C_1 : |z-1| = 2, \quad C_2 : |z-i| = 2,$$

$$\text{c) } \int_C \frac{\cos^2(z)}{(z-\frac{\pi}{4})^4} dz \quad C : |z-1| = 1,$$

$$\text{d) } \int_{C_k} \frac{e^{\pi z}}{z^3 - iz^2} dz \quad k = 1, 2, \quad C_1 : |z| = \frac{1}{2}, \quad C_2 : |z| = 2,$$

Exercise 2:

- a) Find the Laurent series of the function

$$f(z) = \frac{1}{z^2 - 8z + 25}$$

at $z_0 = 4 - 3i$ $f(z)$ in the neighborhood of $z^* = 5i$.

- b) Given

$$f(z) = \frac{1}{z^2 - 2iz + 3},$$

find the Laurent series of the function f at $z_0 = 0$ which converges to $f(2i)$ in the neighborhood of $z^* = 2i$.

Hand in: 20. - 24.6.22