Fachbereich Mathematik der Universität Hamburg **SoSe 2022**
Prof.Dr.I.Gasser
Dr. H. P. Kiani
M. Bänsch

Complex functions for Engineering Students

Sheet 4 (Homework)

Exercise 1:

a) Give a Möbius transform that satisfies

$$T(0) = 2i,\, T(4) = 0,\, T(8) = \infty\,.$$

b) (i) Determine the images of the following lines while using the mapping T from part a). Explain your results.

A) $g_1 = \{z \in \mathbb{C}^* \,:\, \mathrm{Im}\,(z) = 0\}\,.$

B) $g_2 = \{z \in \mathbb{C}^* \,:\, \mathrm{Im}\,(z) = 8 - \mathrm{Re}\,(z)\}\,.$

C) $g_3 = \{z \in \mathbb{C}^* \,:\, \mathrm{Re}\,(z) = \mathrm{Im}\,(z)\}\,.$

(ii) Onto which set is the interior of the triangle with the corners $0,\, 8,\, 4+4i$ mapped? Sketch image and domain in the complex plane!

Exercise 2:

To solve a potential problem, the area outside the two circles

$\tilde{K}_1 := \left\{z \in \mathbb{C} \,:\, |z - \frac{5}{2}| \,\leq\, \frac{3}{2}\right\}$, and

$\tilde{K}_2 := \left\{z \in \mathbb{C} \,:\, |z + \frac{5}{2}| \,\leq\, \frac{3}{2}\right\}$

is to be mapped onto the interior of a circular disk around the origin. Present an adequate mapping.

Exercise 3:

Let i be the imaginary unit and $z = x + iy,\ x, y \in \mathbb{R}$.

a) For which $k, l \in \mathbb{R}$ is the function

$$f : \mathbb{C} \to \mathbb{C}\,,\ f(z) := (x^3 + kxy^2) + i \cdot \left(lx^2y - y^3\right)$$

complex differentiable in every point $\in \mathbb{C}$?

b) Given the function

$$u(x + iy) = \operatorname{Re}(f(x + iy)) = 3\cos(4x)e^{4y}\,.$$

i) Show that the function u is harmonic.

ii) Determine all conjugated harmonic functions v to u, so that all functions v, for which $f = u + iv$, are complex differentiable everywhere in $\mathbb{C}$.

Hand in: 16.- 20.5.22