Fachbereich Mathematik der Universität Hamburg **SoSe 2022**
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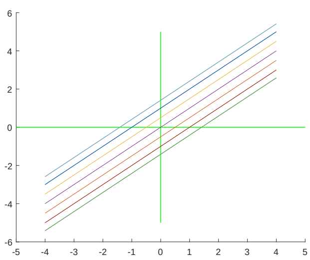
Complex functions for Engineering Students

sheet 3 (Homework)

Exercise 1:

Give a function that maps the strip
$S := \left\{ z \in \mathbb{C} \,:\, \mathrm{Re}\,(z) - \sqrt{2} \,<\, \mathrm{Im}\,(z) \,<\, \mathrm{Re}\,(z) + \sqrt{2} \right\}$



to the circular ring $R := \{ z \in \mathbb{C} : 1 < |z| < 2 \}$. The function is supposed to use z directly and not use its imaginary or real part.

Hint: Transform S to a strip parralel to an axis $\tilde{S}$ first.

Exercise 2: Given the set $R = \{z \in \mathbb{C} : \frac{1}{4} < |z| < \frac{e^3}{4}, \operatorname{Re}(z) > 0\}$,

as well as the mapping

$$f(z) = e^{-i\frac{\pi}{2}} \cdot \ln(4z),$$

where ln is the principal value of the complex logarithm,

a) Sketch the set R in the complexen plane.

b) Determine the image of R obtained with the mapping f.

Exercise 3) (4+3+3 Punkte)

a) For solving two potential problems, the following two transformations are to be executed:

(i) The boundary of the elliptical disk

$$E := \left\{ z = x + iy \in \mathbb{C} : \frac{16x^2}{25} + \frac{16y^2}{9} \le 1 \right\},$$

so $\mathbb{C}\backslash E$, is to be mapped to the outside of the unit circle $K_1 := \{z \in \mathbb{C} : |z| \le 1\}$.

(ii) The area between the two hyperbola branches given by $z = x + iy$ with

$$\frac{4x^2}{3} - 4y^2 = 1 \iff \frac{x^2}{\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

is to be mapped to a sector of the form

$$S := \{z \in \mathbb{C} : \phi_1 < \arg(z) < \phi_2\}.$$

Give adequate transformations.

b) Does your method for solving part a)i) also work for the ellipse

$$E := \left\{ z = x + iy \in \mathbb{C} : \frac{x^2}{25} + \frac{y^2}{9} \le 1 \right\} ?$$

Hint: Inverse of the Joukowski function.