Fachbereich Mathematik der Universität Hamburg **SoSe 2022**
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Complex functions for Engineering Students

Sheet 2 (Homework)

Exercise 1:

Let i be the imaginary unit. Find all complex solutions for the following equations

$$\text{a) } e^{3z} - \frac{i}{e^z} = 0 \qquad \text{bzw.} \qquad \text{b) } \frac{1}{\sqrt{2}}(1+i)\,z^3 = i\,.$$

Exercise 2:

For the $\mathbb{R}^2$, there is a affine linear transformation form any arbitrary rectangle to any arbitrary parallelogram. Check whether the square

$$Q := \{z \in \mathbb{C}, z = x + iy,\ x, y \in [-\sqrt{2}, \sqrt{2}],\ i^2 = -1\}$$

can be transformed (affine linear) to parallelograms with the following corners in $\mathbb{C}$ and if so give an adequate transformation.

a) $\binom{-1}{-i}, \binom{3}{-i}, \binom{3}{i}, \binom{-1}{i}$,

b) $\binom{-1}{-i}, \binom{3}{-i}, \binom{3}{3i}, \binom{-1}{3i}$, c) $\binom{1}{-i}, \binom{2}{2i}, \binom{-1}{i}, \binom{-2}{-2i}$,

d) $\binom{0}{-2i}, \binom{2}{0}, \binom{0}{2i}, \binom{-2}{0}$, e) $\binom{2}{0}, \binom{4}{2i}, \binom{2}{4i}, \binom{0}{2i}$.

Hint: Sketches can be very beneficial.

Exercise 3: (Please read the hints at the end of the exercise)

Given a transformation $w = f(z) := \dfrac{1}{z}$ mit $z \neq 0$.

a) Find the images of

   (i) the ray $\arg(z) = \varphi_0$,

   (ii) the line $\mathrm{Re}(z) = x_0$, so that $z + \overline{z} = 2x_0$,

   (iii) the line $\mathrm{Im}(z) = y_0$.

b) Find the image of the circle $|z - \frac{i}{2}| = \frac{1}{2}$ without $z = 0$.

Hints:

1) Insert $z = \frac{1}{w}$ into the equations which describe the preimage and rearragne the equations as to find out which sets are described in the space of the image.

2) The equation $|z - c| = r$ describes a circle around c with radius r. Be aware that there is the following equivalence that allows for a use without absolute values:

$$|z - c| = R \iff (z - c)\,\overline{(z - c)} = R^2$$

$$\iff (z - c)\,(\overline{z} - \overline{c}) = R^2$$

$$\iff z\overline{z} - z\overline{c} - c\overline{z} + c\overline{c} = R^2 .$$

Hand in: 18.4.22 - 22.4.21