**Mathematics Department** **SoSe 2022**
Prof. Dr. I. Gasser

# Exam Complex functions

**06. September 2022**

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters each in the designated fields following. These entries will be stored on data carriers.

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| **Surname:** | | | | | | | | | | | | | | | | |
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| **BP:** | AIW | CI CS | ET | EUT | GES | IN IIW | MB | MTB MEC | SB | |
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I was instructed about the fact that the required test performance will only be assessed if the TUHH examination office can assure my official admission before the exam's beginning.

(Signature)

| **Task no.** | **Points** | **Evaluater** |
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| **1** | | |
| **2** | | |
| **3** | | |

$\sum$ =

**Task 1) [5 points]**

a) For which $k \in \mathbb{R}$ is the function

$$f : \mathbb{C} \to \mathbb{C}\,,\ f(z) := \left(\mathrm{Re}\,(z)\right)^2 - \left(\mathrm{Im}\,(z)\right)^2 + k \cdot \mathrm{Im}\,(z) + 2i \cdot \mathrm{Re}\,(z) \cdot [\mathrm{Im}\,(z) + 1]$$

complex differentiable in every point in $\mathbb{C}$?

b) In which points in $\mathbb{C}$ does the function

$$g : \mathbb{C} \to \mathbb{C}, \qquad g(z) := z \cdot e^z$$

preserve angles?

**Task 2) [7 points]**

a) Determine a Möbius transform $T : \mathbb{C}^* \to \mathbb{C}^*$, $T(z) := \dfrac{az+b}{cz+d}$ that satisfies

$$T(i) = 0\,, \qquad T(\infty) = 2\,, \qquad T(-1) = \infty\,.$$

b) Which generalized circles in $\mathbb{C}$ are mapped onto lines by $T$?

c) Determine the images of the following generalized circles of T from part a).
$K :=$ real axis,
$\tilde{K} := \{z \in \mathbb{C} : |z| = 1\}$.

**Task 3: (8 points)**

Let $\Gamma := \{z(t) = 2i + 5 \cdot e^{it} \,|\, t \in [0, 2\pi]\}$ be the circle with radius 5 around $2i$ which is traversed once (positively)

Calculate the following path integrals.

a) $\displaystyle\int_\Gamma \frac{z^2}{z-6}\,dz$ .

b) $\displaystyle\int_\Gamma \frac{z^2}{(z-2i)(z+i)}\,dz$ .

c) $\displaystyle\int_\Gamma \frac{z^2}{(z+i)^2}\,dz$ .

d) $\displaystyle\int_\Gamma \overline{(z-2i)}\,dz,$ where $\overline{z}$ is the complex conjugate of $z$.