Fachbereich Mathematik der Universität Hamburg **SuSe 2025**
Prof. Dr. J. Struckmeier
Dr. H. P. Kiani

Differential Equations II for Engineering Students

Homework sheet 6

Exercise 1:

a) Solve the initial value problem

$$\begin{aligned} u_{tt} &= u_{xx}, && \text{on } \mathbb{R}^2\,, \\ u(x,0) &= 2\sin(4\pi x) && x \in \mathbb{R}\,, \\ u_t(x,0) &= \cos(\pi x) && x \in \mathbb{R}\,. \end{aligned}$$

b) Consider the problem

$$\begin{aligned} u_{tt} &= 9u_{xx}, \quad \text{for } x \in \mathbb{R},\ t > 0\,, \\ u(x,0) = u_0(x) &= \begin{cases} 2 & -1 \le x \le 1\,, \\ 0 & \text{otherwise,} \end{cases} \\ u_t(x,0) &= 0. \end{aligned}$$

Sketch the solution obtained by d'Alembert's formula for

$t = 0,\ \frac{1}{6},\ \frac{1}{3},\ \frac{2}{3},\ 1\,.$

Exercise 2:

We are looking for an approximation of the solution to the following problem

$$\begin{aligned} u_{tt} &= u_{xx} && x \in (0, 2\pi),\ t > 0, \\ u(x,0) &= \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \frac{3\pi}{2} \\ x - 2\pi & \frac{3\pi}{2} < x < 2\pi \end{cases} \\ u_t(x,0) &= 0 && x \in (0, 2\pi) \\ u(0,t) = u(2\pi,t) &= 0 && t > 0 \end{aligned}$$

Sketch the 2π-periodic continuation of the initial data for $x \in [-2\pi, 4\pi]$.

Determine an approximation $\tilde{u}$ to the solution u of the problem using the first three terms of the Fourier series.

Check which boundary and initial conditions are already fulfilled by this approximate solution.

Exercise 3:

Consider the initial boundary value problem

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\begin{aligned}
u_{tt} - 4u_{xx} &= e^{-t}\left(1 - \frac{x}{3}\right) && x \in (0,\, 3),\, t > 0,\\
u(x,0) &= 1 + 2\sin(\pi x) && x \in [0,3],\\
u_t(x,0) &= \frac{x}{3} && x \in (0,3), \qquad (1)\\
u(0,t) &= e^{-t} && t \geq 0,\\
u(3,t) &= 1 && t \geq 0.
\end{aligned}
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Introduce a suitable function v in order to convert the problem into an initial boundary value problem with homogeneous boundary conditions for v.

Give the differential equation and the initial conditions for v.

Discussion: 07.07.-10.07.2025