Fachbereich Mathematik der Universität Hamburg **SuSe 2025**
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Differential Equations II for Engineering Students

Work sheet 5

Exercise: (See lecture pages 85-90)

We are looking for the solution to the initial boundary value problem (IBVP)

$$\begin{aligned} u_t - u_{xx} &= e^{-t}\sin(2x) + 1 && x \in (0,\pi)\,, t \in \mathbb{R}^+, \\ u(x,0) &= \frac{1}{2}\sin(2x) && x \in (0,\pi)\,, \\ u(0,t) &= f(t) = t && t \in \mathbb{R}^+ \\ u(\pi,t) &= g(t) = t && t \in \mathbb{R}^+\,. \end{aligned}$$

a) Homogenize the boundary conditions by using the function

$$v(x,t) = u(x,t) - \left[f(t) + \frac{x}{L}\left(g(t) - f(t)\right) \right]$$

with $L = \pi$ and replacing the u–expressions with corresponding v–expressions.

b) Solve the following initial boundary value problem analogously to the procedure in the lecture

$$\begin{aligned} v^*_t - v^*_{xx} &= 0 && x \in (0,\pi)\,, t \in \mathbb{R}^+, \\ v^*(x,0) &= \frac{1}{2}\sin(2x) && x \in (0,\pi)\,, \\ v^*(0,t) &= v^*(\pi,t) = 0 && t \in \mathbb{R}^+\,. \end{aligned}$$

c) Solve the initial boundary value problem

$$\begin{aligned} v^{**}_t - v^{**}_{xx} &= e^{-t}\sin(2x) && x \in (0,\pi)\,, t \in \mathbb{R}^+, \\ v^{**}(x,0) &= 0 && x \in (0,\pi)\,, \\ v^{**}(0,t) &= v^{**}(\pi,t) = 0 && t \in \mathbb{R}^+\,. \end{aligned}$$

using the ansatz

$$v^{**} = \sum_{k=1}^{m} a_k(t)\sin(kx)\,, \qquad a_k(0) = 0$$

d) Give the solution to the initial boundary value problem for u.

Discussion: 23.06.- 26.06.2025