

Differential Equations II for Engineering Students

Homework sheet 5

Exercise 1:

Using a suitable product ansatz, solve the following Dirichlet boundary value problem for the Laplace equation on the circle $r^2 = x^2 + y^2 \leq 9$.

$$\begin{aligned} r^2 u_{rr} + r u_r + u_{\varphi\varphi} &= 0 & 0 \leq r < 3 \\ u(3, \varphi) &= \cos^2(\varphi) & \varphi \in \mathbb{R}. \end{aligned}$$

Hints:

- See lecture pages 85-88.
- To solve Euler's equation $r^2 \cdot w''(r) + ar \cdot w'(r) + b \cdot w(r) = 0$ use the ansatz $w(r) = r^k$.
- It holds: $\cos^2(\varphi) = \frac{1}{2} (1 + \cos(2\varphi))$.

Exercise 2:

- a) Using a product ansatz, derive a series representation for the solution of the following Neumann problem.

$$\begin{aligned} u_t &= u_{xx}, & 0 < x < 1, t > 0, \\ u(x, 0) &= g(x), & 0 < x < 1, \\ u_x(0, t) &= u_x(1, t) = 0 & t > 0. \end{aligned}$$

- b) Solve the initial boundary value problem a) with $g(x) = 3 + 4 \cos(2\pi x)$.

Submission deadline: 27.06.2025