Fachbereich Mathematik der Universität Hamburg **SuSe 2025**
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Differential Equations II for Engineering Students

Homework sheet 4

Exercise 1: See lecture page 52.

Determine the type of the following partial differential equations

a) $2u_{xx} - 8u_{xy} + 8u_{yy} + u_y = u\,,$

b) $2u_{xy} + u_{yy} + xu_x = \cos(y)\,,$

c) $3u_{xx} + 2u_{xy} + u_{yy} = 0\,,$

d) $u_{xx} + e^x u_{yy} + \sin(x)(u_x + u_y) = y + x\,,$

e) $(x^2 + y^2)u_{xx} + 2(x + y)u_{xy} + u_{yy} = 0\,.$

Exercise 2: See lecture pages 59-70.

Let u be a harmonic function in $\Omega := \{\binom{x}{y} \in \mathbb{R}^2 : x^2 + y^2 < 4\}$. Determine the value of $u(0,0)$ for

a) $u(x,y) = \dfrac{x + y + 1}{4}$ on the boundary of $\Omega = \partial\Omega$ using the Poisson integral representation of the solution.

b) $u(x,y) = x^2 y + 2$ on $\partial\Omega$, using the mean value property of harmonic functions.

c) $u(x,y) = x^2 - y^2$ on $\partial\Omega$, using the uniqueness property of the solution.

d) $u(x,y) = x^2 + y^2$ on $\partial\Omega$, without calculation, using the maximum/minimum principle.

Submission deadline: 13.06.2025