Differential Equations II for Engineering Students Work sheet 2

Exercise 1

Compute the solutions of the following initial value problems for u(x,t):

a)

$$u_t + \frac{1}{2}u_x = 0, \qquad x \in \mathbb{R}, t \in \mathbb{R}^+,$$

$$u(x,0) = 2\sin(x), \qquad x \in \mathbb{R}.$$

b) (Exam SuSe17, Ex.2a)

$$u_t + \frac{1}{2}u_x = -4(u+1), \qquad x \in \mathbb{R}, t \in \mathbb{R}^+,$$

$$u(x,0) = 2\sin(x), \qquad x \in \mathbb{R}.$$

Exercise 2:

Determine the solution u(x, y) to the following differential equation

$$xu_x + \frac{y}{2}u_y = u\,,$$

that satisfies the condition $u(1, y) = 1 + y^2$, $y \in \mathbb{R}$. Exercise 3: (only for people who compute fast)

Given the following initial value problem

$$u_t + 3u \cdot u_x = 0, \qquad x \in \mathbb{R}, \ t \in \mathbb{R}^+$$
$$u(x, 0) = \begin{cases} 0 & \forall x \le 0\\ \frac{1}{3} & \forall x > 0 \end{cases}$$

- a) Write down the system of characteristic equations.
- b) Are the characteristics straight lines?
- c) Draw the characteristics through the points $(x_k, 0) := (k, 0)$ for $k \in \{-3, -2, -1, 0, 1, 2, 3\}$. Compute the values of the solution along these characteristics.
- d) Using parts a)-c), can you obtain the values of u(x,t) in the points (-1,2), (1,2) and (3,2)?

Discussion: 05.05.-08.05.2025