Fachbereich Mathematik der Universität Hamburg **SuSe 2025**
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Differential Equations II for Engineering Students

Homework sheet 2

Exercise 1: **[5 Points]**

Compute the solution to the following initial value problem for $u(x,t)$:

$$\begin{aligned} u_t - \sin(t)\, u_x &= \cos(t)\,, & x \in \mathbb{R},\ t \in \mathbb{R}^+, \\ u(x,0) &= \exp(-x^2) = e^{-x^2}, & x \in \mathbb{R}\,. \end{aligned}$$

Exercise 2: **[6= 2+1+2+1 points]**

Given are the following differential equations for $u(x,t)$, $u : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$

A) $u_t + 20\, u_x = 21u\,.$
B) $u_t + 20u\, u_x = 21\,.$
C) $u_t - 5u^2\, u_x = 0.$
D) $u_t + 5(x+1)\, u_x = 0.$

with the initial condition

$u(x,0) = u_0(x), \qquad x \in \mathbb{R}\,,$

where $u_0 : \mathbb{R} \to \mathbb{R}$ is a monotonically increasing and continuously differentiable function.

For which of the differential equations A, B, C, D do the following statements i) and/or ii) hold for the solution of the associated initial value problem?

i) The solution is constant along the characteristics.
ii) The characteristics are straight lines.

Explain your answers. Note that you don't have to compute any solutions!

Exercise 3:

Determine a continuous "solution" $u(x,t)$ for the following initial boundary value problem

$$\begin{aligned} u_t + u_x &= x, & x,t > 0 \\ u(x,0) &= x, & (x \geq 0) \\ u(0,t) &= t, & (t \geq 0) \end{aligned}$$

using the method of characteristics. To do this, determine a solution u_I for the initial condition $u(x,0) = x$ and a solution u_B for the boundary condition $u(0,t) = t$ and continuously compose these solutions. Is the solution obtained in this way partially differentiable for all $x, t \geq 0$?

Submission deadline: 09.05.2025