Fachbereich Mathematik der Universität Hamburg **SuSe 2025**
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Differential Equations II for Engineering Students

Work sheet 1

Exercise 1:

We are looking for solutions of the heat equation in one spatial dimension:

$$u_t - cu_{xx} = 0$$

with a fixed parameter $c \in \mathbb{R}^+$ (thermal conductivity / diffusion coefficient).

Show that for any number $\omega \in \mathbb{R}$ and for any $k \in \mathbb{Z}$

$$u_k(x,t) = \sin(\ k\omega x\ ) \cdot e^{-ck^2\omega^2 t}$$

is a solution of the differential equation .

Obviously, a so-called product ansatz: $u(x,t) = q(t) \cdot p(x)$ leads to solutions of the heat equation.

Exercise 2:

We now consider the Telegraph Equation.

A signal of the periodic voltage

$$U(0,t) = U_0 \cos(\omega t) \qquad t \geq 0$$

is fed in at the starting point $x = 0$ of a very long transmission cable. We are looking for the signal voltage $U(x,t)$ of the output signal for $x > 0, t > 0$. One obtains U as the solution of the differential equation

$$U_{tt} - c^2 U_{xx} + (\alpha + \beta) U_t + \alpha\beta U = 0\,.$$

Where α, β, c are positive parameters determined by the problem. A temporally periodic input signal leads to the expectation of a temporally periodic output signal after a certain transient phase. In addition we expect that

$$U(x,t) \quad \text{is bounded for} \quad x \to \infty\,.$$

a) Show that an approach that combines a local dampening (factor e^{-kx}) with a temporally periodic behavior (i.e. cosine/sine in t) and allows for a linear location-dependent phase shift leads to success. For example:

$$U(x,t) := e^{-kx} \cdot \Big(\delta \cos(\mu t - \gamma x) + \tilde{\delta} \sin(\tilde{\mu} t - \tilde{\gamma} x)\Big)$$

For the sake of simplicity, set $\alpha = \beta = c = 1$.

Hint: $a^2 b^2 + a^2 - b^2 - 1 = (a^2 - 1)(b^2 + 1)\,.$

b) **(Only for very fast students)**

Show that the product approach $U(x,t) = w(x) \cdot v(t)$ is not successful here.

Again, choose $\alpha = \beta = c = 1$.

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