**Mathematics Department**
Prof. Dr. J. Struckmeier

**SuSe 2025**

# Exam Differential Equations II

**27. August 2025**

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters each in the designated fields following. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exams beginning in retrospect.

(Signature)

| **Task** | **Points** | **Evaluator** |
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| **1** | | |
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**Problem 1:  [7 Points]**

Compute the solution of the following initial value problem for $u(x,t)$:

$$\begin{aligned} u_t + (2t+1)\, u_x &= -ut\,, \qquad && x \in \mathbb{R},\, t \in \mathbb{R}^+, \\ u(x,0) &= \sin(2x)\,, && x \in \mathbb{R}\,. \end{aligned}$$

**Problem 2: [4 Points]**

Determine the entropy solution of the differential equation

$$u_t + (f(u))_x = 0$$

with the flux function

$$f(u) = \left(\frac{u-1}{3}\right)^2$$

and the initial condition

$$u(x,0) = \begin{cases} 2 & x \leq 0, \\ -1 & 0 < x. \end{cases}$$

Note: Only the solution for the given initial values is required. You don't need to give solutions for general initial values!

**Problem 3: [2 points]**

Let $u(x, y)$ be the solution to the following boundary value problem

$$\begin{aligned} \Delta u &= u_{xx} + u_{yy} = 0, \quad \text{in } \Omega := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2,\, 0 < x < 20,\, 0 < y < 25 \right\}, \\ u(x, y) &= 10, \quad \text{on } \partial\Omega\,. \end{aligned}$$

Determine the solution $u$ without calculation. Justify your answer.

**Problem 4: [7 points]**

a) Consider the initial boundary value problem

$$\begin{aligned} u_t - u_{xx} &= \frac{x}{\pi}\cos(t) && \text{for } x \in (0,\pi), t > 0, \\ u(x,0) &= 1 - \frac{x}{\pi} + 2\Big(\sin(x) - \sin(3x)\Big) && \text{for } x \in [0,\pi], \\ u(0,t) &= 1, && \text{for } t > 0, \\ u(\pi,t) &= \sin(t) && \text{for } t > 0. \end{aligned}$$

Introduce a suitable function $v$ in order to convert the problem into an initial boundary value problem with homogeneous boundary conditions for $v$.

Determine the differential equation and the initial conditions for $v$.

b) Solve the following initial boundary value problem:

$$\begin{aligned} v_t - v_{xx} &= 0 && \text{for } x \in (0,\pi), t > 0, \\ v(x,0) &= 2\sin(x) - 2\sin(3x) && \text{for } x \in [0,\pi], \\ v(0,t) &= 0, \quad v(\pi,t) = 0 && \text{for } t > 0. \end{aligned}$$

c) Give the solution to the initial boundary value problem from part a).