Fachbereich Mathematik der Universität Hamburg **SuSe 2024**
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Differential Equations II for Engineering Students

Work sheet 6

Exercise 1:

a) We consider the initial value problem

$$\begin{aligned} u_t - 2u_{xx} &= \frac{1}{2}x\cos(t) && \text{for } x \in (0,2), t > 0, \\ u(x,0) &= 20\sin(2\pi x) + 24\sin(4\pi x) + \frac{x}{2} && \text{for } x \in [0,2], \\ u(0,t) &= 0, \quad u(2,t) = 1 + \sin(t) && \text{for } t > 0. \end{aligned}$$

Transfer the problem into an initial value problem with homogeneous boundary data via a suitable homogenization of the boundary values.

b) Solve the following initial value problem

$$\begin{aligned} v_t - 2v_{xx} &= 0 && \text{for } x \in (0,2), t > 0, \\ v(x,0) &= 20\sin(2\pi x) + 24\sin(4\pi x) && \text{for } x \in [0,2], \\ v(0,t) &= 0, \quad v(2,t) = 0 && \text{for } t > 0. \end{aligned}$$

c) Determine the solution to the initial value problem from part a).

Exercise 2:

a) Determine the solution of the following initial value problem:

$$\begin{aligned} u_{tt} - u_{xx} &= 0 && x \in (0, \frac{\pi}{2}),\ t > 0, \\ u(x,0) &= \begin{cases} x & x \in [0, \frac{\pi}{4}], \\ \frac{\pi}{2} - x & x \in [\frac{\pi}{4}, \frac{\pi}{2}], \end{cases} \\ u_t(x,0) &= 2\sin(4x) && x \in [0, \frac{\pi}{2}], \\ u(0,t) &= u(\frac{\pi}{2}, t) = 0 && t > 0. \end{aligned}$$

b) Consider the initial value problem

$$\begin{aligned}
&u_{tt} - 4u_{xx} = e^{-t}\left(1 - \frac{x}{3}\right) && x \in (0,\,3),\ t > 0,\\
&u(x,0) = 1 + 2\sin(\pi x) && x \in [0,3],\\
&u_t(x,0) = \frac{x}{3} && x \in [0,3], \qquad (1)\\
&u(0,t) = e^{-t} && t > 0,\\
&u(3,t) = 1 && t > 0.
\end{aligned}$$

for $u = u(x,t)$. Which initial value problem do we obtain after a suitable homogenization of the boundary values?

Discussion: 01.07.- 05.07.2024