Fachbereich Mathematik der Universität Hamburg **SuSe 2024**
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Differential Equations II for Engineering Students

Homework sheet 6

Exercise 1:

a) Determine the solution to the initial value problem

$$\begin{aligned} u_t - 4u_{xx} &= 0, \qquad 0 < x < 1,\ t \in \mathbb{R}^+, \\ u(x,0) &= x - \sin\left(\frac{\pi}{2}x\right), \qquad 0 \le x \le 1 \\ u(0,t) &= u(1,t) = 0, \qquad t > 0. \end{aligned}$$

Hint for the integration: $\sin(\alpha x)\sin(\gamma x) = \cos(\alpha x - \gamma x) - \cos(\alpha x + \gamma x)$.

b) We consider the following initial value problem for $u = u(x,t)$:

$$\begin{aligned} u_t - 2u_{xx} &= -2xe^{-2t} + \sin(2\pi x), \qquad && x \in (0,\,2),\ t > 0, \\ u(x,0) &= 1 + \frac{x}{2} + 3\sin(3\pi x), && x \in [0,2], \\ u(0,t) &= 1, \qquad u(2,t) = 2e^{-2t}, && t \ge 0. \end{aligned} \tag{1}$$

(i) Show that the homogenization of the boundary values with suitably defined function v gives the following problem:

$$\begin{aligned} v_t - 2v_{xx} &= \sin(2\pi x), \qquad && x \in (0,\,2),\ t > 0, \\ v(x,0) &= 3\sin(3\pi x), && x \in [0,2], \\ v(0,t) &= v(2,t) = 0, && t \ge 0. \end{aligned} \tag{2}$$

(ii) Solve the initial value problem (2) from part i).

Exercise 2: Solve the initial value problem:

$$
\begin{aligned}
u_{tt} \;-\; 4u_{xx} \;&=\; 3\sin(2\pi x)\cdot e^{-2t} && x \in (0,1),\, t>0\\
u(x,0) \;&=\; u_0(x) = \sin(\pi x) \;+\; 4\sin(2\pi x) && x \in [0,1],\\
u_t(x,0) \;&=\; v_0(x) = 0 && x \in [0,1],\\
u(0,t) \;&=\; 0 && t>0,\\
u(1,t) \;&=\; 0 && t>0,
\end{aligned}
$$

Hint: Insert the ansatz

$$u(x,t) = \sum_{k=1}^{\infty} q_k(t)\sin(k\omega x), \qquad \omega = \frac{\pi}{1}$$

in the differential equation. You will obtain an ODE for q_k.

The initial values will give initial values for q_k.

Hand in: 01.07.- 05.07.2024