Fachbereich Mathematik der Universität Hamburg **SuSe 2024**
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Differential Equations II for Engineering Students

Homework sheet 5

Exercise 1:

(a) Let

$$\Omega_2 = \{(x,y)^\top \in \mathbb{R}^2 \; : \; 1 < x^2 + y^2 < 4\}.$$

Determine the solutions of

$$\begin{cases} \Delta u = 0 & \text{on} & \Omega_2, \\ u(x,y) = 1 & \text{for} & x^2+y^2 = 1, \\ u(x,y) = 3 & \text{for} & x^2+y^2 = 4. \end{cases}$$

Is the solution unique?

(b) Let

$$\Omega_3 = \{(x,y,z)^\top \in \mathbb{R}^3 \; : \; 1 < x^2 + y^2 + z^2 < 4\}.$$

Determine the solutions of

$$\begin{cases} \Delta u = 0 & \text{on} & \Omega_3, \\ u(x,y,z) = 1 & \text{for} & x^2+y^2+z^2 = 1, \\ u(x,y,z) = 3 & \text{for} & x^2+y^2+z^2 = 4. \end{cases}$$

Is the solution unique?

Exercise 2: We are looking for a solution of the Laplace equation $\Delta v(x,y) = 0$ in a rotationally symmetrical area, for example in a circular ring. The area can then be better described using polar coordinates. This is done as follows

$x = r\cos\phi$, $y = r\sin\phi$, and

$v(x(r,\phi), y(r,\phi)) = u(r,\phi)$.

Show that for $r \neq 0$ the following equivalence holds:

$$r^2 u_{rr} + r u_r + u_{\varphi\varphi} = 0 \iff r^2 \left(v_{xx} + v_{yy}\right) = 0\,.$$

Exercise 3:
Determine all rotationally symmetrical solutions of the following boundary value problem

$$\begin{aligned} \Delta u(x,y) &= -\frac{1}{\sqrt{x^2+y^2}} \quad \text{for } 1 < x^2+y^2 < 9, \\ u(x,y) &= 1 \quad \text{on } x^2+y^2 = 1, \\ u(x,y) &= 2 \quad \text{on } x^2+y^2 = 9. \end{aligned}$$

Hand in: 17.06.-21.06.2024