Fachbereich Mathematik der Universität Hamburg **SuSe 2024**
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Differential Equations II for Engineering Students

Work sheet 3

Exercise 1:

We consider the initial value problem

$$\begin{aligned} u_t + 4t\,u_x &= 3\,, && x \in \mathbb{R},\, t \in \mathbb{R}^+, \\ u(x,0) &= \sin(2x), && x \in \mathbb{R}\,. \end{aligned}$$

a) First, derive the characteristic differential equations and determine their general solution.

b) Continue with computing the solutions $u(x,t)$ of the initial value problem.

Exercise 2:

Compute the solution to the following initial value problem for $u(x,t)$:

$$\begin{aligned} u_t + \tfrac{1}{2}\,u_x &= -4\,u, && x \in \mathbb{R},\, t \in \mathbb{R}^+, \\ u(x,0) &= \tfrac{2\sin(x)}{1+x^2}, && x \in \mathbb{R}\,. \end{aligned}$$

Exercise 3: (only for particularly fast students)
Given the following initial value problem

$$u_t + 3u \cdot u_x = 0, \qquad x \in \mathbb{R},\, t \in \mathbb{R}^+$$

$$u(x,0) = \begin{cases} 0 & \forall x \le 0 \\ \frac{1}{3} & \forall x > 0 \end{cases}$$

a) Write down the system of characteristic equations.

b) Are the characteristics straight lines?

c) Draw the characteristics through the points
$(x_k, 0) := (k, 0)$ for $k \in \{-3, -2, -1, 0, 1, 2, 3\}$.
Compute the values of the solution along these characteristics.

d) Using parts a)-c), can you obtain the values of $u(x,t)$ in the points $(-1,2), (1,2)$ and $(3,2)$?

Discussion: 13.05.-17.05.2024