# **Differential Equations II for Engineering Students**

## Homework sheet 3

#### Exercise 1:

Compute the solution to the following initial value problem for u(x,t):

$$u_t - \sin(t) u_x = \cos(t), \qquad x \in \mathbb{R}, t \in \mathbb{R}^+,$$
$$u(x, 0) = \exp(-x^2) = e^{-x^2} \qquad x \in \mathbb{R}.$$

- a) First, derive the characteristic differential equations and determine their general solution.
- b) Continue with computing the solutions u(x,t) of the initial value problem.

#### Exercise 2:

Solve the initial value problem

$$\begin{aligned} u_t + 3u_x + y^2 u_y &= 0, \qquad x, y \in \mathbb{R}, t \in \mathbb{R}^+, \\ u(x, y, 0) &= \frac{\cos(x)}{1 + y^2} \qquad x, y \in \mathbb{R}. \end{aligned}$$

#### Exercise 3:

Given are the following differential equations for  $u(x,t), u: \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ 

- A)  $u_t + 20 u_x = 21 u$ .
- **B)**  $u_t + 20u u_x = 21$ .
- C)  $u_t 5u^2 u_x = 0.$
- **D)**  $u_t + 5(x+1)u_x = 0.$

with the initial condition

$$u(x,0) = u_0(x), \qquad x \in \mathbb{R},$$

where  $u_0: \mathbb{R} \to \mathbb{R}$  is a monotonically increasing and continuously differentiable function.

For which of the differential equations A, B, C, D do the following statements i) and/or ii) hold for the solution of the associated initial value problem?

i) The solution is constant along the characteristics.

ii) The characteristics are straight lines.

### Explain your answers. Note that you don't have to compute any solutions!