

Differential Equations II for Engineering Students

Work sheet 1

Exercise 1: (Repetition of DGL I)

- a) Let λ be any fixed real number. Determine a real representation of the general solution to the differential equation

$$y''(t) - \lambda y(t) = 0.$$

- b) Let L be another fixed positive real number. Determine all solutions to the boundary value problem

$$y''(t) - \lambda y(t) = 0 \quad y(0) = y(L) = 0.$$

For which $\lambda \in \mathbb{R}$ does the boundary value problem have nontrivial solutions?

The λ -values for which there exist non-trivial solutions (i.e. solutions that are not constantly equal to zero) are called eigenvalues of the problem. The corresponding solutions are called eigenfunctions.

Remark: *The solutions to this eigenvalue problem will be needed again and again during the semester!*

Exercise 2:

Determine the appropriate real Fourier series for the following functions:

- a) Odd $2L$ -periodic continuation of

$$f : [0, 1[\rightarrow \mathbb{R}, \quad f(t) = \sin(4\pi x) + 2 \sin(6\pi x) \quad L = 1.$$

- b) Even $2L$ -periodic continuation of

$$f : [0, \frac{\pi}{2}[\rightarrow \mathbb{R}, \quad L = \frac{\pi}{2} \quad \text{und}$$

$$f(t) = \begin{cases} 2, & 0 \leq t < \frac{\pi}{4}, \\ 0, & \frac{\pi}{4} \leq t < \frac{\pi}{2}. \end{cases}$$

Determine the first four non-vanishing summands of the Fourier series.

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