Mathematics Department **WiSe 2024/25**
Prof. Dr. T. Schmidt

Exam: Differential Equations II

04. March 2025

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number each in block letters in the designated fields following. These entries will be stored on data carriers.

Surname:																
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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

Signature:

Task no.	**Points**	**Evaluator**
1		
2		
3		
4		

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Exercise 1: [5 Points]

Given the following initial value problem

$$u_t + t \cdot u_x = 1, \qquad \text{for} \qquad x \in \mathbb{R},\ t > 0,$$

$$u(x, 0) = \cos(x), \qquad \text{for} \qquad x \in \mathbb{R}.$$

a) state the characteristic equations for this problem and determine their solutions,

b) solve the initial value problem.

Exercise 2: [6 Points]

For $u(x,t)$ the following initial value problem is given:

$$u_t + u \cdot u_x = 0, \qquad x \in \mathbb{R},\, t \in \mathbb{R}^+$$

$$u(x,0) = \begin{cases} 2 & x \leq -2, \\ 0 & -2 < x \leq 1, \\ -1 & 1 < x\,. \end{cases}$$

a) Determine the physically reasonable solution of the initial value problem for $t \in [0, t^*)$ with suitably small t^*.

b) Up to which t^* does the solution formula from a) make sense?

c) How can the solution be extended for $t > t^*$ in a physically reasonable way?

Exercise 3: [6 Points]

Determine the solution of the following initial boundary value problem:

$$
\begin{aligned}
& u_{tt} - 9u_{xx} = 0 && 0 < x < 2\,,\ 0 < t\,,\\
& u(x,0) = 5\sin(2\pi x) + 7\sin(3\pi x) && 0 \le x \le 2,\\
& u_t(x,0) = 9\sin(\pi x) && 0 \le x \le 2,\\
& u(0,t) = 0 && 0 \le t,\\
& u(2,t) = 0 && 0 \le t.
\end{aligned}
$$

Exercise 4: [3 Points]

Let $\tilde{u}$ and $\hat{u}$ be solutions of the differential equation

$$u_t - u_{xx} + u = 2, \qquad x \in (0,1),\, t \in \mathbb{R}^+,$$

for $u(x,t)$, which satisfy the boundary conditions

$$u(0,t) = 0, \qquad u(1,t) = \sin(t), \qquad t \in \mathbb{R}^+.$$

a) Is $\tilde{u} + \hat{u}$ a solution of the differential equation?
Justify your answer.

b) Does $\tilde{u} - \hat{u}$ solve the following differential equation

$$u_t - u_{xx} + u = 0, \qquad x \in (0,1),\, t \in \mathbb{R}^+,$$

and satisfy the boundary conditions

$$u(0,t) = 0, \qquad u(1,t) = \sin(t), \qquad t \in \mathbb{R}^+?$$

Justify your answer.