Mathematics Department
Prof. Dr. T. Schmidt

SuSe 2024

Exam Differential Equations II

26. August 2024

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number each in block letters in the designated fields following. These entries will be stored on data carriers.

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Task no.	**Points**	**Evaluator**
1		
2		
3		
4		

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Exercise 1: [5 Points]

Given the following initial value problem

$$u_t + \frac{1}{t+1} \cdot u_x = u \qquad \text{for} \qquad x \in \mathbb{R},\ t > 0,$$

$$u(x,0) = e^{-x} \qquad \text{for} \qquad x \in \mathbb{R},$$

a) state the characteristic equations for this problem and determine their solutions,

b) solve the initial value problem.

Exercise 2: [4+1 Points]

Given the following initial value problem for $u(x,t)$

$$u_t + u \cdot u_x = 0, \qquad x \in \mathbb{R},\, t \in \mathbb{R}^+$$

$$u(x,0) = \begin{cases} 2 & \text{for } x \leq -1, \\ 0 & \text{for } -1 < x \leq 0, \\ 1 & \text{for } 0 < x, \end{cases}$$

a) determine the physically reasonable solution of the initial value problem for $0 < t < 1$.

b) Why does the solution formula from a) only hold for $t < 1$?

Exercise 3: [1+2,5+2,5 Points]

Determine the bounded solution of the following boundary value problems for the Laplace equations: You can give the solutions in cartesian or polar coordinates.

a) $$\begin{cases} \Delta u = 0 & \text{in} \quad \Omega_1 := \{\binom{x}{y} \in \mathbb{R}^2,\, x^2+y^2 < 25\}, \\ u(x,y) = 4 & \text{for} \quad x^2+y^2 = 25. \end{cases}$$

b) $$\begin{cases} \Delta u = 0 & \text{in} \quad \Omega_1 := \{\binom{x}{y} \in \mathbb{R}^2,\, x^2+y^2 < 25\}, \\ u(x,y) = u(r\cos(\phi), r\sin(\phi)) = 3\sin(2\phi) & \text{for} \quad x^2+y^2 = 25. \end{cases}$$

c) $$\begin{cases} \Delta u = 0 & \text{in} \quad \Omega_2 := \{\binom{x}{y} \in \mathbb{R}^2,\, 1 < x^2+y^2 < 25\} \\ u(x,y) = 4 & \text{for} \quad x^2+y^2 = 1, \\ u(x,y) = 2 & \text{for} \quad x^2+y^2 = 25. \end{cases}$$

Exercise 4: [3+1 Points]

Given the following initial boundary value problem

$$\begin{aligned} u_t - 16u_{xx} &= 4\cos(t)\Big(1-\frac{x}{2\pi}\Big) && \text{for } x \in (0,2\pi), t>0, \\ u(x,0) &= \frac{x}{2\pi} && \text{for } x \in [0,2\pi], \\ u(0,t) &= 4\sin(t), \quad u(2\pi,t) = 1 && \text{for } t>0. \end{aligned}$$

a) transfer this problem into an initial boundary value problem with homogeneous boundary data for a function $v(x,t)$ via a suitable homogenization.

   Write down the new initial boundary value problem (differential equartion, initial values, boundary values).

b) Write down a solution v for the initial boundary value problem with homegeneous boundary data from part a) without any computations. What is the corresponding solution u of the originial problem?