Fachbereich Mathematik der Universität Hamburg **SoSe 2023**
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Differential Equations II for Engineering Students

Homework sheet 7

Exercise 1: Solve the initial boundary value problem:

$$\begin{aligned}
u_{tt} - 4u_{xx} &= 3\sin(2\pi x)\cdot e^{-2t} \qquad && x\in(0,1),\ t>0\\
u(x,0) &= u_0(x) = \sin(\pi x) + 4\sin(2\pi x) && x\in[0,1],\\
u_t(x,0) &= v_0(x) = 0 && x\in[0,1],\\
u(0,t) &= 0 && t>0,\\
u(1,t) &= 0 && t>0,
\end{aligned}$$

Hint: Insert the ansatz

$$u(x,t) = \sum_{k=1}^{\infty} q_k(t)\sin(k\omega x), \qquad \omega = \frac{\pi}{1}$$

into the differential equation. You get the ordinary differential equation for q_k. The initial conditions provide the initial data for the q_k.

Exercise 2: *(So that you don't get the idea that you can solve all linear differential equations of second order with a simple product ansatz.)*

At the starting point $x = 0$ of a very long transmission cable there is a signal of the periodic voltage

$$U(0,t) = U_0\cos(\omega t) \qquad t\ge 0,\ \omega>0\,.$$

We are looking for the signal voltage $U(x,t)$ of the output signal for $x > 0, t > 0$. One obtains U as the solution of the so-called telegrapher's equations.

$$U_{tt} - c^2U_{xx} + (\alpha+\beta)U_t + \alpha\beta U = 0\,.$$

Here α, β, c are construction-related positive parameters of the problem. A temporally periodic input signal leads to expect a temporally periodic output signal after a certain settling phase. In addition, one requires

$$U(x,t) \quad \text{bounded for} \quad x\to\infty\,.$$

a) Show that the product ansatz $U(x,t) = w(x)\cdot v(t)$ is not working here!

b) Try the solution ansatz that combines "local damping"(factor e^{-kx}) with a time-periodic progression (i.e. cosine/sine for t) and allows a linear, location-dependent phase shift. So for example

$$U(x,t) := e^{-kx}\cdot\Big(\delta\cos(at-bx) + \tilde\delta\sin(\tilde a t-\tilde b x)\Big)$$

Choose for example $\alpha=\beta=c=1$.