Differential Equations II for Engineering Students

Work sheet 6

Exercise:

Given the following initial boundary value problem for $u = u(x,t)$:

$$u_t - u_{xx} = e^{-t}\sin(2x) + 1 \qquad x \in (0,\pi)\,, t \in \mathbb{R}^+,$$

$$u(x,0) = \frac{1}{2}\sin(2x) \qquad x \in (0,\pi)\,,$$

$$u(0,t) = u(\pi,t) = t \qquad t \in \mathbb{R}^+\,.$$

a) Perform the homogenization of the boundary values.
   What initial boundary value problem does one obtain after homogenizing the boundary values?

b) Solve for following the initial and boundary data:

   (i)
   $$v_t^* - v_{xx}^* = 0 \qquad x \in (0,\pi)\,, t \in \mathbb{R}^+,$$

   $$v^*(x,0) = \frac{1}{2}\sin(2x) \qquad x \in (0,\pi)\,,$$

   $$v^*(0,t) = v^*(\pi,t) = 0 \qquad t \in \mathbb{R}^+\,.$$

   and
   $$v_t^{**} - v_{xx}^{**} = e^{-t}\sin(2x) \qquad x \in (0,\pi)\,, t \in \mathbb{R}^+,$$

   $$v^{**}(x,0) = 0 \qquad x \in (0,\pi)\,,$$

   $$v^{**}(0,t) = v^{**}(\pi,t) = 0 \qquad t \in \mathbb{R}^+\,.$$

c) Provide the solution to the initial boundary value problem from part a).

Discussion: 26.06 - 30.06.2023