Fachbereich Mathematik der Universität Hamburg **SuSe 2022**
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Differential Equations II for Engineering Students

Homework sheet 6

Exercise 1: (Exam, Prof. Behrens 2022, 7 Points)

a) Given the initial boundary value problem

$$\begin{aligned} u_t - 5u_{xx} &= \frac{\pi x}{4}\sin(\pi t) && \text{for } x \in (0,4), t > 0, \\ u(x,0) &= 2\sin(\pi x) + 3\sin(2\pi x) && \text{for } x \in [0,4], \\ u(0,t) = 0, \quad u(4,t) &= 1 - \cos(\pi t) && \text{for } t > 0. \end{aligned}$$

Transform the problem into an initial boundary value problem with homogeneous boundary data using a suitable homogenization of the boundary conditions.

b) Solve the following initial boundary value problem:

$$\begin{aligned} v_t - 5v_{xx} &= 0 && \text{for } x \in (0,4), t > 0, \\ v(x,0) &= 2\sin(\pi x) + 3\sin(2\pi x) && \text{for } x \in [0,4], \\ v(0,t) = 0, \quad v(4,t) &= 0 && \text{for } t > 0. \end{aligned}$$

c) Provide the solution to the initial boundary value problem from part a).

Exercise 2:

a) Using a product ansatz, derive the series representation given in lecture 10 (page 18) for the solution of the following Neumann problem.

$$\begin{aligned} u_t &= u_{xx}, && 0 < x < 1,\ t > 0, \\ u(x,0) &= g(x), && 0 < x < 1, \\ u_x(0,t) &= u_x(1,t) = 0 && t > 0. \end{aligned}$$

b) Solve the initial boundary value problem a) with $g(x) = 2\pi x - \sin(2\pi x)$.

Hint: $2\sin(\alpha)\cdot\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta)$.

Abgabe bis: 30.06.2023