Fachbereich Mathematik der Universität Hamburg SuSe 2023

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Differential Equations II for Engineering Students

Work sheet 5

Exercise 1:

Determine the solution to the following problem

$$\Delta v = 0 \quad \text{for } 0 \le x^2 + y^2 < 9,$$

$$v(x, y) = \frac{x}{9}(x - y) \quad \text{on } x^2 + y^2 = 9.$$

Hint:

- *Use polar coordinates and an appropriate product approach.*

- $\cos(2\phi) = 2\cos^2(\phi) - 1,\ \sin(2\phi) = 2\sin(\phi)\cos(\phi)\,.$

Exercise 2:

Solve the following Dirichlet problems

a)

$$\begin{aligned}
\Delta v &= 0 & \text{in } (0,2) \times (0,1),\\
v(x,0) &= \sin(\pi x), & x \in (0,2),\\
v(x,1) &= 0, & x \in (0,2),\\
v(0,y) &= 0, & y \in (0,1)\\
v(2,y) &= 0, & y \in (0,1).
\end{aligned}$$

b)

$$\begin{aligned}
\Delta w &= 0 & \text{in } (0,2) \times (0,1),\\
w(x,0) &= 0, & x \in (0,2),\\
w(x,1) &= -5\sin(2\pi x), & x \in (0,2),\\
w(0,y) &= 0, & y \in (0,1)\\
w(2,y) &= 0, & y \in (0,1).
\end{aligned}$$

c)

$$\begin{aligned}
\Delta u &= 0 & \text{in } (0,2) \times (0,1),\\
u(x,0) &= 6\sin(\pi x), & x \in (0,2),\\
u(x,1) &= 5\sin(2\pi x), & x \in (0,2),\\
u(0,y) &= 0, & y \in (0,1)\\
u(2,y) &= 0, & y \in (0,1).
\end{aligned}$$

Discussion: 12-16.06.2023