Fachbereich Mathematik der Universität Hamburg **SuSe 2023**
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Differential Equations II for Engineering Students

Work sheet 4

Exercise 1:

Given the partial differential equation

$$3u_{xx} + 8u_{xt} - 3u_{tt} = 0 \quad \text{for } x \in \mathbb{R}, t > 0$$

a) Determine the type of the differential equation (hyperbolic, parabolic or elliptic).

b) Transform the differential equation into the diagonal form $\alpha \cdot \tilde{u}_{\eta\eta} + \beta \cdot \tilde{u}_{\tau\tau} = 0$.

c) How do the new coordinates η, τ depend on the old coordinates t, x?

Exercise 2:

Determine the value of the harmonic in $\Omega := \{\binom{x}{y} \in \mathbb{R}^2 : x^2 + y^2 < 4\}$ function $u(x, y)$ in the point $\binom{0}{0}$ with the boundary data

a) $u(x, y) = \dfrac{x + y + 1}{4}$ on the boundary of $\Omega = \partial\Omega$ using the Poisson integral representation of the solution.

b) $u(x, y) = x^2y + 2$ on $\partial\Omega$, using the mean value property of harmonic functions.

c) $u(x, y) = x^2 - y^2$ on $\partial\Omega$, using the uniqueness property of the solution.

d) $u(x, y) = x^2+y^2$ on $\partial\Omega$, without calculation, using the maximum/minimum principle.

Dates: 29.05.-02.06.2023